INVESTIGATING FIFTH-GRADE STUDENTS' FUNCTIONAL THINKING PROCESSES THROUGH A GAME-BASED LEARNING ACTIVITY

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

INVESTIGATING FIFTH-GRADE STUDENTS' FUNCTIONAL THINKING PROCESSES THROUGH A GAME-BASED LEARNING ACTIVITY

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The aim of this study was to investigate fifth-grade students' generalization and representation processes of functional relationships through a game-based learning activity. This study was carried out with four students selected from the fifth grade in a state village middle school in Mardin. In this school, where the researcher worked as a teacher, a Functional Thinking Test was applied to two classes, and participants were selected based on pre-test responses and the ability to express themselves. A pre-interview was conducted to better understand the students' solutions in the test. A digital game was designed by the researcher, which includes functional relationships such as recursive patterns, covariational thinking, and one-to-one correspondence, and a game interview was carried out to be applied to the students after the game. After the participants interacted with the game, the same test was applied as a post-test, and then the final interview was held. Students' answers regarding generalization and representation of functional relationships were assessed through correctness and strategies using qualitative data analysis. The results of the research showed a significant improvement in the students' generalization and representation processes of functional relationships after the interaction with the game. Another important finding was that the students used symbols and expressed the functional relationships meaningfully.

Keywords: Early Algebra, Functional Thinking, Game-based Learning, Recursive Pattern, Covariational Thinking, Correspondence Thinking

ÖZ

BEŞİNCİ SINIF ÖĞRENCİLERİN FONKSİYONEL DÜŞÜNME SÜREÇLERİNİN OYUN TEMELLİ BİR ÖĞRENME ETKİNLİĞİ İLE İNCELENMESİ

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Bu çalışmanın amacı, beşinci sınıf öğrencilerinin oyun tabanlı bir öğrenme etkinliği ile fonksiyonel ilişkileri genelleme ve temsil etme süreçlerini incelemektir. Bu çalışma, Mardin ilinde bir devlet köy ortaokulunda beşinci sınıftan seçilen dört öğrenci ile gerçekleştirilmiştir. Araştırmacının öğretmen olarak görev yaptığı bu okulda, iki sınıfa Fonksiyonel Düşünme Testi uygulanmış katılımcılar ön test cevapları ve kendilerini ifade etme becerilerine göre seçilmiştir. Öğrencilerin testteki çözüm yollarını daha iyi anlamak amacıyla ön görüşme yapılmıştır. Araştırmacı tarafından yinelemeli örüntüler, kovaryasyonel düşünme, bire bir eşleyerek düşünme gibi fonksiyonel ilişkileri içeren bir dijital oyun tasarlanmış ve oyunun hemen ardından öğrencilere uygulanmak üzere bir görüşme gerçekleştirilmiştir. Katılımcılar oyunla etkileşime girdikten sonra aynı test son test olarak uygulanmış ve ardından son görüşme yapılmıştır. Öğrencilerin fonksiyonel ilişkileri genelleme ve temsil etme konusundaki cevapları nitel veri analizi kullanılarak doğruluk ve kullanılan stratejiler açısından değerlendirilmiştir. Araştırma sonuçları, öğrencilerin oyunla etkileşimi sonrasında fonksiyonel ilişkileri genelleme ve temsil süreçlerinde önemli bir gelişme olduğunu göstermiştir. Bir diğer önemli bulgu ise öğrencilerin sembolleri kullanmaları ve fonksiyonel ilişkileri anlamlı bir şekilde ifade etmeleridir.

Anahtar Kelimeler: Erken Cebir, Fonksiyonel Düşünme, Oyun Tabanlı Öğrenme, Yinelemeli Örüntü, Kovaryasyonel Düşünme, Bire Bir Eşleyerek Düşünme

To all my loved ones

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ABBREVIATIONS

Functional Thinking Test	FTT
Ministry of National Education	MoNE
National Council of Teachers of Mathematics	NCTM

CHAPTER 1

INTRODUCTION

Since the beginning of the 21st century, early algebra has been at the center of studies on algebra and has an important place in the study of many researchers. Some researchers (e.g., Blanton & Kaput, 2005; 2011; Carpenter et al., 2003; Ryan & Williams, 2007; Stephens et al., 2015) argue that with the appropriate teaching and environment, algebraic thinking can and should be fostered at an early age along with arithmetic thinking. Blanton and Kaput (2005) revealed as "algebraic reasoning needs to develop over a long period in students' mathematical experience, beginning in the early grades and engaging most mathematical topics" (p. 100). Therefore, early learning of algebra and related actions should be included to comprehend further algebraic structures and to increase students' readiness for these concepts. That is to say, early algebra lays the foundation for students to construct ways of thinking that will help them comprehend algebra later on. According to Chimoni et al. (2019), familiarizing elementary school students with algebra at an early age is supposed to help children transition from concrete, arithmetic thinking to the more complicated, abstract algebraic thinking required for high school mathematics onwards. The extension of arithmetic prepares the ground for the growth of early algebraic reasoning. Algebraic thinking takes place from kindergarten to grade 12 in the Common Core State Standards for Mathematics (CCSSM, 2010).

An important and appropriate way to introduce the concept of algebra to students in elementary school is to develop students' functional thinking abilities. Functional thinking contributes to the development of algebraic thinking. Functional thinking is regarded as being essential to studying mathematics in later school years because functions are used to simulate many real-world applications (Wilkie & Clarke, 2015). Students can become aware of its essential structure and properties by gradually gaining the knowledge necessary to define and explain the functional relationships between the co-variate quantities (Chimoni et al., 2019).

Functional thinking is of great importance in making sense of different mathematical concepts. Different methods and techniques can be used to help students gain functional thinking processes. Game-based learning can be used as an important educational method to provide students with such processes. "Game-based learning' broadly refers to the use of video games to support teaching and learning" (Perrotta et al., 2013, p. 6). Integrating games into the lesson can be an encouraging method for students in mathematics lessons, which are generally perceived as difficult lessons and are unwilling to attend. Previous research provides evidence about the positive effects of the integration of games into lessons. One aspect is that algebra learning can become more engaging, fun, and meaningful with game-based learning activities (e.g., Annetta et al., 2009; Ke, 2008; Ya-Ting, 2012). This means that incorporating these activities into the teaching of algebra and mathematics assists in better learning. The other one is that thanks to this integration, students are more motivated to participate in the lesson and develop deeper comprehension levels (Allsop et al., 2013). Also, some views indicate that games enhance students' cognitive functions, such as critical and strategic thinking (Kirriemuir & McFarlane, 2004).

In the transition from arithmetic to algebra, middle school students have trouble understanding some algebraic structures and the underlying meaning of some algebraic procedures or operations. One of them is the difficulty of comprehending that variables can represent more than one value (e.g., Kaput, 1998; Kieran, 2007; Yerushalmy & Chazan, 2002). The other ones are that the students have trouble constructing a correspondence rule (Canadas et al., 2016; Carraher et al., 2008), and they have difficulty generalizing and representing functional relationships (English & Warren, 1998; McGregor & Stacey, 1995; Warren et al., 2013).

Students also have motivation problems to engage in solving functional or algebraic problems. According to Leroy and Bressoux (2016), many students develop negative attitudes toward mathematics during their first few years in elementary school. Many of these students struggle with math by the 5th grade simply since they do not learn best through the popular method of rote memorization. That's why students find math boring and difficult. Because they think like this, when they see a math problem, they approach it with prejudice and think they cannot solve it. Some researchers also claim that many students dislike or fear mathematics because of how the subject is taught in the classroom (Boaler, 2014; Brady & Bowd, 2005; Metje et al., 2007; Scarpello, 2007). It may be necessary to increase the interest and motivation of the students in the lesson and to teach them using different methods and techniques to overcome this. Thus, familiarizing elementary students with early algebra is supposed to facilitate the transition from concrete and arithmetic thinking to more abstract algebraic thinking. Learning algebra can be more engaging, fun, and meaningful with game-based learning activities so that students can grasp mathematical situations easily and develop deeper comprehension. It provides ongoing learning opportunities beyond essential skills through knowledge application to different situations, communication, information evaluation, collaboration, and problem-solving (Salpeter, 2003). Some researchers also have suggested that computer math games could enhance mathematics performance (Ke & Grabowski, 2007; Moreno, 2002; Rosas et al., 2003) and could positively affect students' motivation in math (Lopez-Morteo & Lopez, 2007; Rosas et al., 2003).

1.1 The Purpose of the Study

The aims of this study are to investigate the fifth-grade students' development of functional thinking processes within game-based learning activities and to explore students' abilities for making generalizations, identifying variables, and representing functional relationships.

1.2 Research Questions

1) How do fifth-grade students' functional thinking processes develop after their engagement in the learning process with game-based learning activities?

a. How do fifth-grade students' generalization process of the functional relationships develop with the implementation of game-based learning activities?

b. How do fifth-grade students' representation process of the functional relationships develop with the implementation of game-based learning activities?

1.3 Significance of the Study

Algebraic thinking involves recognizing and analyzing patterns, representing relationships, generalizing, and analyzing how things change. It also is about reasoning, using notations, and calculation of unknown and numbers (Radford, 2011). During my teaching experience, I had a chance to observe the students' solution ways, or strategies and their reasoning process about algebra. I realized that students had problems understanding the structures of algebra, developing algebraic reasoning, and performing algebraic operations related to them. There are also many studies in the literature that support these troubles. Elementary and middle school students have difficulties comprehending algebraic structures and solving related problems (e.g., Kaput, 1998; Kieran, 2007; Yerushalmy & Chazan, 2002). Additionally, the students have trouble constructing a correspondence rule (Canadas et al., 2016; Carraher et al., 2008), and they have difficulty generalizing and representing functional relationships (English & Warren, 1998; McGregor & Stacey, 1995; Stephens et al., 2017; Warren et al., 2013). To overcome these, the students should encounter algebra early and must be familiarized with the algebraic structure so that their algebraic reasoning can be developed and their difficulties can be reduced.

There are limited studies on students' functional thinking processes and the development of these processes in Turkey. Tanışlı (2011) examined the fifth-grade students' functional thinking ways. Türkmen and Tanışlı (2019), on the other hand, revealed the functional relationships generalization skills of third, fourth, and fifth-grade students. Another study on the functional thinking of fifth-grade students was conducted by Akın (2020) by designing a functional thinking intervention consisting of 5 five lesson plans. In addition, Ozturk et al. (2020) investigated the effect of an early algebra approach on the functional thinking skills of third-grade students. None of these studies used game-based learning as an intervention method.

Among the teaching techniques that are currently highly popular is game-based learning. Integrating the game into the lessons can be effective for students' learning process. According to Bakan and Bakan (2018), educational games help students participate in the learning process while also improving their comprehension of the course requirements. Another piece of evidence supporting the use of educational games to assist and improve mathematics learning outcomes is provided by Pratama and Setyaningrum (2018). Similar arguments were made in the literature that playing these games enhances students' interest in the course (e.g., Malone, 1981), they were more motivated and engaged to participate in the lesson, and they developed more positive attitudes toward math learning (e.g., Annetta et al., 2009; Ke, 2008; Malone, 1981; Ya-Ting, 2012), they can positively affect deeper comprehension levels, critical thinking and problem-solving (e.g., Allsop et al., 2013; Kirriemuir & McFarlane, 2004; Kolovou & Heuvel-Panhuizen, 2010). Thus, game-based learning exercises can make algebra teaching more engaging, enjoyable, and meaningful for children so they can comprehend mathematical situations better.

Limited studies focused on functional thinking processes, and the development of these processes through game-based learning. Therefore, the findings of this research would contribute to the literature investigating early grade students' algebraic thinking and learning processes with game-based learning. Thus, this study aimed to investigate fifth-grade students' development of generalization and representation processes of functional relationships within the game-based activities using a multi-case study design.

1.4 Definition of the Terms

Algebraic thinking or reasoning: "Algebraic reasoning is characterized by the dual abilities, on one hand, to generalize, justify, and express generality within structured symbolic forms, and on the other to use the structure of these symbolic forms to reveal deeper relationships and generalizations" (Kaput & Blanton, 2005, p. 100).

Functional Thinking: "Functional thinking entails (a) generalizing relationships between covarying quantities; (b) representing and justifying these relationships in multiple ways using natural language, variable notation, tables, and graphs; and (c) reasoning fluently with these generalized representations in order to understand and predict functional behavior" (Blanton et al., 2015, p. 512).

Early Algebra: It is defined as algebra in the early grades, which "encompasses algebraic reasoning and algebra-related instruction among young learners—from approximately 6 to 12 years of age" (Carraher & Schliemann, 2007, p. 670).

Recursive pattern: "It involves finding variation within a sequence of values" (Blanton et al., 2011, p. 52).

Covariational thinking: It "involves analyzing how two quantities vary in relation to each other and keeping that variation explicit in the description of the function" (Blanton et al., 2011, p. 52).

Correspondence thinking: It is "a correlation between two quantities expressed as a function rule." (Blanton et al., 2011, p. 53)

Game-based learning: It is "a form of learning packaged with games based on specific plans, programs, tools, and equipment prepared by the teacher, and then students are trained in playing the game to achieve the learning objectives set" (Afikah, 2022, p. 702).

Generalization: As defined in NCTM's Developing Essential Understanding of Mathematical Reasoning K–8 (Lannin et al., 2011), generalizing refers either to identifying commonality across cases or extending commonality beyond the domain of the original pattern.

CHAPTER 2

LITERATURE REVIEW

This study investigates the fifth-grade students' functional thinking processes in game-based learning and explores their ability to generalize and represent functional relationships. In this part, to accomplish these goals, it is crucial first to describe what algebraic thinking and early algebra are. Then, students' understandings of functional thinking and which difficulties and misconceptions elementary and middle students experience regarding functional thinking will follow. After these, what game-based learning is, and the related studies about game-based learning will be provided. Finally, studies focusing on game-based learning and algebra, specifically functional thinking, will be mentioned.

2.1 Algebraic Thinking and Early Algebra

The students' learning and understanding of fundamental concepts of algebra are essential since algebra is a key component in acquiring knowledge of high school mathematics (Rakes et al., 2010). Algebra is defined as " understanding pattern, relations, and functions; representing and analyzing mathematical situations and structures using algebraic symbols; using mathematical models to represent and understand quantitative relationships and analyzing the change in various contexts" (NCTM, 2000, p. 37). Algebraic thinking or reasoning can be defined as "forming generalization from a set of particular instances or experiences, express them within gradually a formal and symbolic system and exploring the concepts of pattern and functions" (Van de Walle et al., 2011, p. 258). The development of algebraic thinking occurs from preschool to high school (Van de Walle et al., 2011). To handle the high school and middle school students' troubles in algebra, prompting them to think algebraically in early grades helps to prevent these difficulties (Cai et al., 2011;

Warren & Cooper, 2008). Additionally, they advise students to focus on algebra and arithmetic during the initial five to six years of primary school. Starting in the early grades and involving most mathematical concepts, algebraic thinking has to develop for an extended period in children's mathematics experiences (Kaput & Blanton, 2005). Thus, to comprehend further algebraic structures and to increase students' readiness for these concepts, early learning of algebra and related actions should be included at elementary school levels.

Since the beginning of the 21st century, early algebra has been at the center of studies on algebra and has an important place in the study of many researchers. Some researchers (e.g., Blanton & Kaput, 2011; Carpenter et al., 2003; Ryan & Williams, 2007) argue that algebraic reasoning should be promoted from an early age along with arithmetic thinking. Chimoni et al. (2019) state that introducing algebra to elementary-level children at an early age is supposed to advance them from concrete, arithmetic thinking to the more complicated and abstract algebraic thinking required for further grade math. The expansion of arithmetic lays the groundwork for the growth of early algebraic reasoning. In this way, students can become aware of its fundamental structure and characteristics. They can also gradually improve the ability to describe and explain functional relationships between covariate quantities. Therefore, early learning of algebra and related actions should be included to comprehend further algebraic structures and to increase students' readiness for these concepts. That is to say, early algebra lays the foundation for students to construct thought processes that could help them to learn algebra later on. This study also aims to investigate fifth grade studens' functional thinking processes to increase students' readiness for these concepts.

2.1.1 Algebra in the National Mathematics Curriculum

Students first encounter the learning area for algebra in the 6th grade in the Grades 1-8 National Curriculum supplied by the Ministry of National Education (MoNE, 2018). Before middle school, there is no algebra learning area; however, some objectives are related to the big ideas of algebra: equivalence and equations, generalized arithmetic, functional thinking, variable, and quantitative reasoning (Blanton et al., 2011). Objectives addressing algebra, specifically functional thinking, in elementary and middle school grades (MoNE, 2018) are shown in Table 2.1.

Table 2.1 Objectives addressing functional thinking in Grades 1-8 (MoNE, 2018) Grades

Grades	Number in the Curriculum	Objective
1st Grade	M.1.2.3.1	Students find the rule of a pattern consisting of objects, a geometric object or figure, and complete the pattern by identifying the missing objects in the pattern.
2nd Grade	M.2.1.1.6	Students identify number patterns that have a constant difference, find the rule of the pattern, and complete the pattern by determining the missing item.
3rd Grade	M.3.1.1.7	Students expand and generate a number of patterns that have a constant difference.
5th Grade	M.5.1.1.3	Students find the required steps of the given number and figure patterns.
6th Grade	M.6.2.1.1	Students write an algebraic expression for the given verbal situation and write a verbal situation for the given algebraic expression.
	M.6.2.1.2	Students compute the value of the algebraic expression for different natural number values that the variable can take.

Table 2.1 (continued)

Grades	Number in the Curriculum	Objective
7th Grade	M.7.2.1.3	Students express the rule of the number patterns using letters and find the asked term of the pattern when the rule was expressed by letters.
	M.7.2.2.2	Students identify linear equations with one unknown and construct a linear equation with one unknown corresponding to the given real-life situations.
	M.7.2.2.3	Students solve equations with unknown.
	M.7.2.2.4	Students solve the problems that require constructing linear equations with one unknown.
8th Grade	M.8.2.2.1	Students solve the problems that require constructing linear equations with one unknown.
	M.8.2.2.2	Students identify the coordinate system with its characteristics and show the coordinates.
	M.8.2.2.3	Students express how one of the variables changes in relation to the other using a table and an equation when there is a linear relationship between the variables.
	M.8.2.2.4	Students draw a graph of linear relationships.
	M.8.2.2.5	Students formulate equations, tables, and graphs for real-life situations involving linear relationships and interpret them.

2.2 Functional Thinking and a Framework for Functional Thinking

Functional thinking involves "generalizing relationships between covarying quantities, expressing those relationships in words, symbols, tables, or graphs, and reasoning with these various representations to analyze function behavior" (Blanton & Kaput, 2011, p. 47).

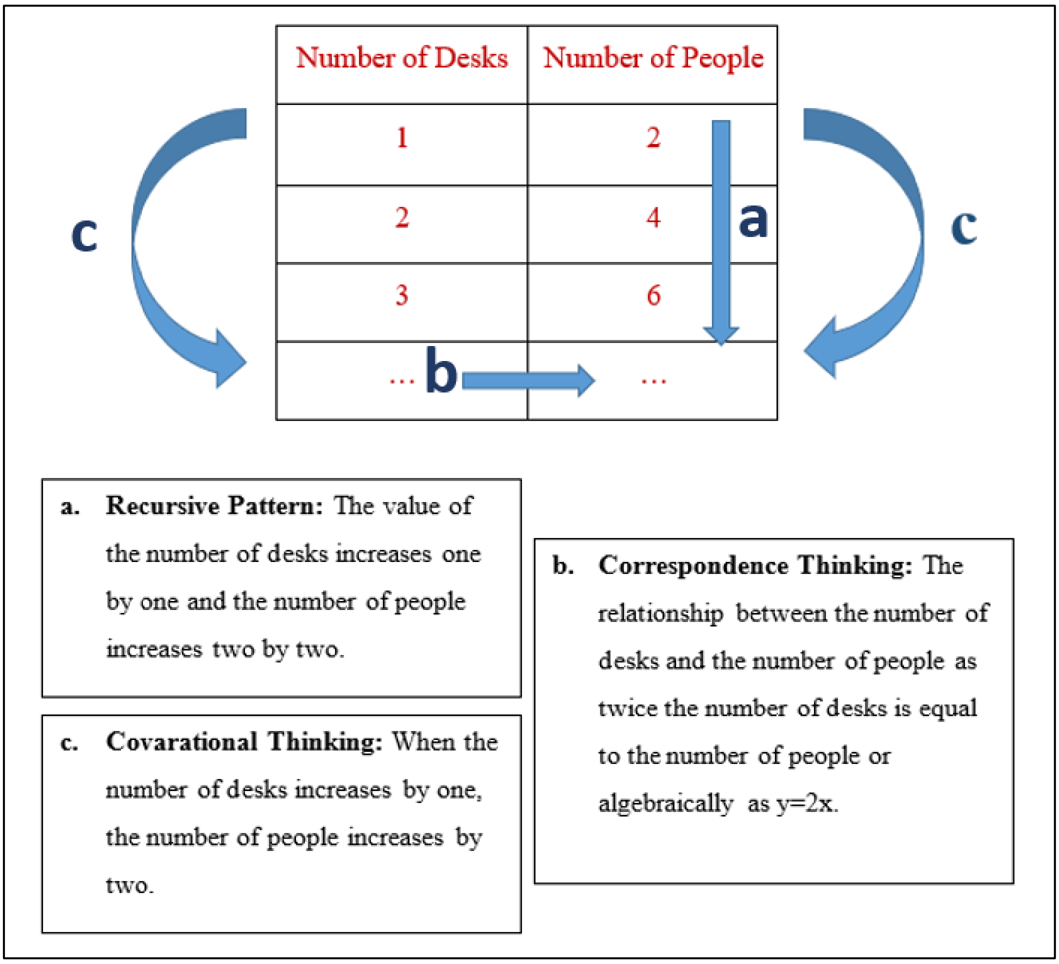


Figure 2.1 Example of the Types of Functional Relationships

There are three forms of functional thinking: recursive pattern, covariational thinking, and correspondence relationships. A recursive pattern is about getting variation within a sequence of values. Students engaged in recursive thinking to look for a relationship in a single string of numbers. Covariate thinking relies on evaluating how two quantities change simultaneously and maintaining that variation as an apparent, dynamic aspect of a function's formulation. Students engaged in covariational thinking to examine how two quantities varied in respect to one another (Blanton & Kaput, 2011; Stephens et al., 2017). The correspondence relationship is founded on the discovery of a correlation between variables. Students engaged in covariational thinking can often generalize the relationship between the two quantities in question (Confrey & Smith, 1991; Smith, 2008). In this study, for example, the Brady task problem was used to get students to think about the number of people and the number of desks. Figure 2.1 demonstrates the three types of functional relationships for the Brady task problem.

According to NCTM (2000), mathematics instruction should assist all students in "creating and using representations to organize, record, and communicate mathematical ideas; selecting, applying, and translating among mathematical representations to solve problems; and using representations to model and interpret physical, social, and mathematical phenomena" (p. 67). Blanton et al. (2011) indicated that in the elementary grades, acquiring an awareness of various representational forms such as tables, graphs, and variables helps students build key abilities that can assist a deep analysis of relationships among these representations in the middle grades. It can also facilitate the more formal study of proportional relationships as a specific instance of linear functions in the middle grades. Thus, understanding these representations and their relationships is critical. Considering these, this study aimed to examine students' generalization and representation processes of functional relationships. For this purpose, the framework of Stephens et al. (2017) was used. This will be discussed in detail in the next section.

2.2.1 Levels of Sophistication of Functional Thinking

In a three-year longitudinal study, Stephens et al. (2017) examined elementary students’ progress in generalizing and representing functional relationships. As part of a comprehensive early algebra approach, they focused on a context that elicits the functional thinking of 3-5th grade students and representations such as words, symbols, tables, pictures, and graphs. This teaching sequence included a total of 18 lessons designed as small group work and whole class discussions. Of these, 7 lessons were taught for each of the 3rd and 4th grades, and six lessons were taught for the 5th grades related to functional thinking. The lesson plans were developed considering the fundamental concepts and big ideas of algebraic thinking. Students in grade 3 used representations, such as coordinate graphs, to work on the recursive, covariational, and correspondence relationships associated with the y=mx and y=x+b functions. The quadratic functional relationships $y=x^2$ and $y=x^2 + b$ were emphasized in Grade 4. In Grade 5, students learned about exponential and quadratic functions. A coding scheme based on the levels of sophistication reflecting students' generalization and representation of functional relationships was used to analyze the students' responses (see Figure 2.2).

The framework of this study was adapted from the research of Stephens et al. (2017). Levels of Sophistication Describing Grades 3-5 Students' Generalization and Representation of Functional Relationships was presented in Figure 2.2.

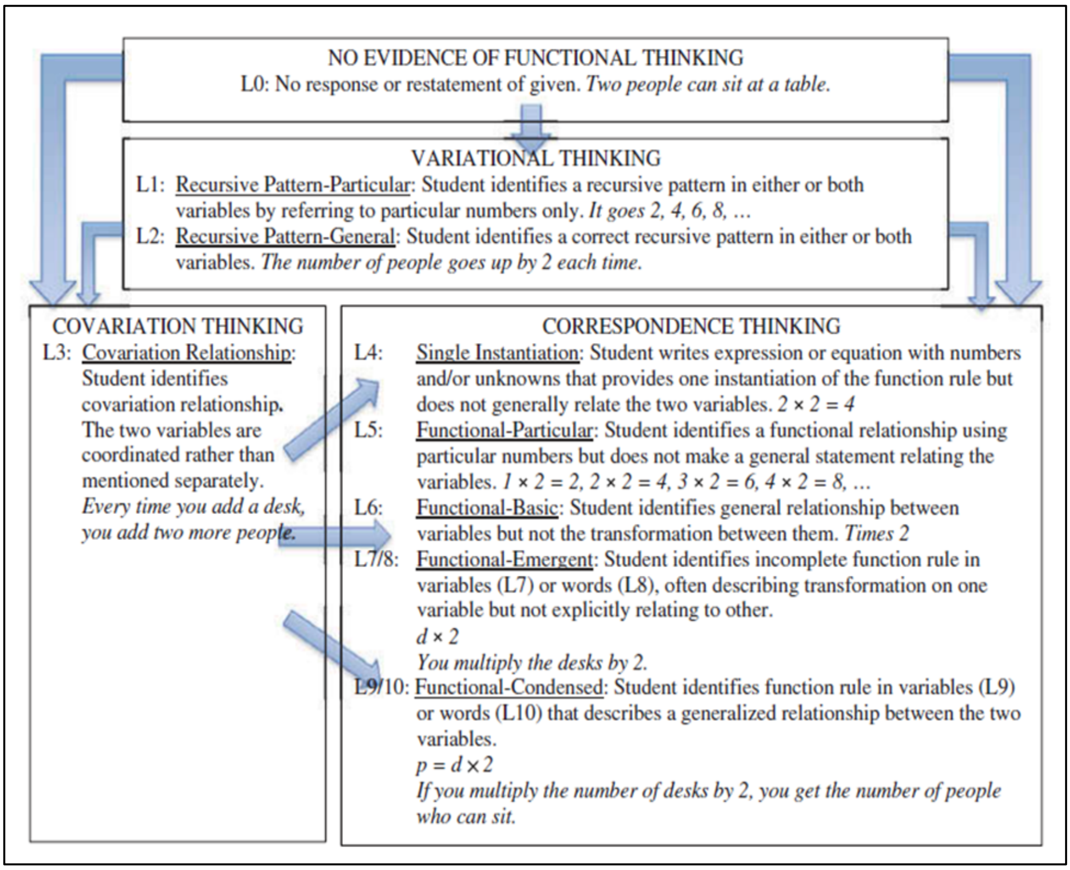


Figure 2.2 Levels of Sophistication Describing Grades 3-5 Students' Generalization and Representation of Functional Relationships
Note. Levels of Sophistication Describing Grades 3-5 Students' Generalization and Representation of Functional Relationships, by Stephens et al., 2017, Mathematical Thinking and Learning, 19:3, p. 153

Stephens et al. (2017) defined the relationships between quantities in three categories recursive, correlational, and corresponding thinking. These categories, as seen in Figure 2.2, were divided into sub-levels according to the generalization and representation ways used by the students while answering the questions. For example, if students used variables to write the function rule, they stated Functional-Emergent in variables and Functional-Condensed in variables; if they used words to write the function rule, they supplied Functional-Emergent in words and Functional-Condensed in words. They located the word categories at a higher level since students could define the function rules in variables instead of words more easily.

Instead of going to the next level right away, students could progress by skipping these levels.

2.2.2 Representations of Functional Thinking

Urena et al. (2022) examined the strategies and representations of sixth-grade primary school students with no prior algebraic training to generalize functional relationships. Students were given problems including functions, divisibility, number operations, measurement, and constructions that point to various linear functional relationships in various situations. Students are encouraged to draw pictorial representation, organize it inductively, and devise various solutions throughout the problem-solving stage. When the students indicated a general rule connecting the variables by a recognized regularity, they were considered to have expressed generalization. They discussed the many classification schemes used to categorize students' representations of generalization, including verbal, symbolic, and multiple. According to their findings, students used various strategies and representations, such as verbally, symbolically, or multiple, to generalize. The most widely used of these is the correspondence strategy. They stated that students were able to use symbolic as well as verbal representations while generalizing.

Pinto and Canadas (2021) investigated how third-year students relate to and represent the relationships between variables while working on functional problems involving a function of type y=mx+n that they have not studied before. The categories used in the research to analyze the students' answers are presented in Table 2.2. They created seven sub-categories to analyze students' forms used to represent functional relationships. The results showed that students first proved their correspondence relationship verbally and numerically to express this functional relationship. They also stated that, even though students were unfamiliar with operating with such kinds of problems, a significant portion of students went beyond arithmetic computations, discovering relationships between variables. Furthermore, some students generalized practically using natural language, whereas others recognized the same pattern for distinct specific values but could not represent generalization.

Table 2.2 Analysis Category for Representation from the Study of Pinto and Canadas (2021)

Category	Sub-category
Representation	2.1. Natural language
	2.2. Manipulative
	2.3. Pictorial
	2.4. Numerical
	2.5. Algebraic notation
	2.6. Tabular
	2.7. Multiple representations

**All the possible combinations of more than one of the types of representations listed

Note. Retrieved from Functional Relationships Evidenced and Representations used by Third Graders within a Functional Approach to Early Algebra by Pinto and Canadas, 2021, *International Journal of Science and Mathematics Education*, 20(6), pp. 1183-1202.

2.3 Studies Related to Functional Thinking

Some studies on students' generalization and representation of functional relationships are mentioned in this part of the study. At the same time, some studies examining the development processes of students with game-based learning are also included.

Ferrara and Sinclair (2016) conducted pattern generalization activities to familiarize the 2nd and 3rd-grade students with early algebra. These tasks were connected to recursively seeking sameness in a pattern and speculating about function-based interactions connecting variables. They aimed for a new materialistic approach that aims to define phenomena in terms of material entanglement, which includes not only the children and the instructor but also numbers, variables, operations, gestures, words, arrangements, and objects. As a result, the researchers enounced that the discourse approach that focuses on pattern generalization improved the functional reasoning of early grade students.

Warren et al. (2006) also focused on functional thinking by investigating the development of about nine-year students' functional thinking during four lesson teaching experiments. These lessons were created to help students develop mental images for exploring the usage of function tables by concentrating on the connection between input and output numbers. This study revealed that the students were not only able to develop functional thinking but also, they could communicate their thoughts both symbolically and verbally.

Besides, Warren and Cooper (2008) profound that adolescents' algebra difficulties can be prevented by teaching actions and thinking at an early age. They designed two lesson teaching experiments for about eight-year students and teachers. The planned assignment involved expanding the pattern and investigating the relationship between the location number and the patterns. The results indicated that young students cannot only think about the connection between two sets of data but can also express this relationship in a fairly abstract way.

There is also a study by Blanton et al. (2017) focused on investigating variable quantity and children's variable notation of functional relationships. They used an instructional sequence to explore first-grade children's learning and development of variables and variable notation in functional relationships, and they reported on their progress in understanding these concepts. The findings indicated that children's perceptions of symbolic notation shifted from not knowing how to use a letter to represent a variable quantity to interpreting a letter as representing something unknown but not inherently mathematical. According to the findings of this study, young children can think sophisticatedly about variable quantities and variable notation. They also suggest that starting formal school with long-term, maintained experiences with variable and variable notation may help adolescents overcome difficulties or misconceptions.

One of the most important components of this study is its focus on generalization. Mathematical topics and activities rely heavily on generalization. Generalization implies a deliberate widening of the range of reasoning or communication beyond the scope of the case or cases under consideration. Generalization also refers to the clear identification of commonalities between events or the raising of reasoning or communication to a level where attention is no longer given to the situations or cases but instead to the patterns, procedures, structures, and relations between them (Kaput, 1999). Mason (2008) emphasized that generalization and symbolization are interconnected processes that begin at a young age. The ability to see connections between variables described by a set of functional rules is a significant issue of generalization, as underlined by functional approaches to algebra learning. Growing patterns have been generalized as one approach to developing the ability to comprehend and represent these connections. The role of generalization has broadened the apparent quality in algebra classes expressing the need to develop algebraic reasoning in generalization activities (Kaput, 1999; Kieran, 2004). Making generalizations is of great importance in developing algebraic understanding. Carpenter and Franke (2001) said that generalizations provide a class with basic mathematical propositions to investigate. They also stated that students express their mathematical thoughts while generalizing number and operation properties. Therefore, students need to learn to generalize across all mathematical disciplines. Kaput indicates that “the heart of algebraic reasoning is comprised of complex symbolization processes that serve purposeful generalization and reasoning with generalizations” (Kaput, 2008, p. 9). Blanton et al. (2011) emphasized a functional approach to early algebra, emphasizing the view of arithmetic operations as functions and allowing pupils to investigate the concept of variable as a variation between quantities. This approach emphasized the need to generalize functional relationships and represent relationships.

Despite the importance of generalization in numerical and arithmetic thinking, its cycles are unknown. Students struggle with generalizing arithmetic and algebraic expressions. Studies have shown that students struggle with making correct generalized conclusions (English & Warren, 1995; Kieran, 1992; Lannin, 2005), taking care of generalizable patterns (Blanton & Kaput, 2002; Lee, 1996), including using generalized expression (Mason, 1996). Lannin’s study demonstrated that during small-group conversations, students seldom explained their generalizations, with some emphasizing specific values rather than general relationships (Lannin, 2005).

Research looking at students’ generalizing practices in algebra settings has additionally distinguished various difficulties. One of these is that students have trouble generalizing patterns that are useful for algebra. Schoenfeld (1985) discussed the technical challenges that some students had while selecting the right mathematical model and determining the appropriate linear relationship. According to Stacey (1989), children who believe a relationship might be true use it without hesitation. Lee and Wheeler (1987) studied with tenth-grade students to investigate their generalization and algebraic thinking process. Their research used generalization problems relating to linear and quadratic patterns. The study led them to the realization that students perceived patterns in every question in a wide range of ways. However, they didn't find much proof that students control their patterns. However, only one of the eight students they interviewed checked whether their model worked. They noticed that there was no student reaction or tendency to compare the formula to the provided proof formula. The second one is that the students have difficulty moving from detecting patterns to generalizing them. According to Stacey and McGregor (1997), students expect that any operation they can imagine or discuss can be expressed in simple mathematics. Therefore, they concluded that students had trouble creating formulas from tables and number patterns. They claimed that rather than rules between two variables, students attempted to seek rules for computing the next number in a sequence. Even though students' difficulties with generalizing have been well reported, extra examination on the most proficient method to advance successful generalizing in classroom settings is required to assist instructors with supporting their students' generalization processes more readily.

Representation is one of the essential points of this study. Kaput and Blanton (2011, p. 8) note that “the connections between different representations help to resolve some of the ambiguity of isolated representations, [so] for concepts to be fully developed, children will need to represent them in various ways.” There are various studies on the representation forms of students. Pinto and Canadas (2021) conducted research with 24 third-graders and 24 fifth-graders, examining the functional relationships demonstrated in the children's responses and their representations. For this, they conducted a Classroom Teaching Experiment in each grade. They assessed the written responses of the students in response to a variety of questions meant to help them generalize the relationships in a problem involving the equations $y = 2x + 6$. They revealed that almost half of the third and most fifth graders confirmed functional relationships in their answers. They discovered that students in higher grades tended to concentrate on relationships between variables. They discovered that third-year students, in contrast, tended to concentrate more on the specifics of arithmetic calculations. They claimed that the students' prior classroom math experiences were likely responsible for these differences. Furthermore, they discovered that natural language was the primary means of generalization in both classes. Fifth graders, unlike third graders, recognized general norms from numerical calculations and articulated them even when they were not expressly asked to. Tanışlı (2011) studied fifth-grade pupils and collected data through task-based interviews. Linear function tables were used to explore functional thinking in the early stages. As a result, it was discovered that when working with the linear function tables, the four fifth-grade students were thinking about covariation. It was also discovered that the students could recognize and generalize the correspondence relationship. The study's findings also offered information on the students' reasoning abilities or alternative methods of thinking in generalizing the correspondence relationship.

In this study, symbols were vital to explaining the relationship between variables. In this context, Blanton et al. (2011) mentioned that the concept of variable has different meanings depending on the context in which it is used. One of them is that it represents a fixed and unknown value. For a given equation to be true, this value must be equal to a constant number. The equations $y + 5 = 8$ and $2x – 3 = x + 1$ could be given as examples of this meaning of the variable. In there, the letters x in the first equation and the letter y in the second equation are unknown values, and their values must be fixed numbers to make the equations true. For instance, the letter y represents the number 3, and the letter x represents the number 4 so that the equations are true. Another role is in Blanton et al. (2011) stated in their work that for an equation to be true, that value must be equal to more than one number that satisfies the equation depending on the other quantity. The equation $y=3x$ can be given as an example of this meaning. There are multiple x and y values for this equation to be true. For example, when the values $x=1$ and $y=3$, or $x=2$and $y=6$ are inserted into this equation, they make the equation true. The point to be noted here is that the values represented by the letters x and y vary depending on each other. Once the number value represented by one is known, the other cannot represent a random number. The value of y should always be three times the value of x.

Studies on students' functional thinking processes and the development of these processes are limited in Turkey. Tanışlı (2011) examined the fifth-grade students' functional thinking ways. The research findings revealed that students could find one-to-one matching relationships and generalize them, as well as covariational thinking skills, through the tables in which they represented linear functions. Türkmen and Tanışlı (2019), on the other hand, in their study to reveal the functional relationships generalization skills of third, fourth, and fifth-grade students, revealed that students have many indicators of functional thinking in the literature. About half of the third-year students and more than half of the fourth- and fifth-year students are at the levels showing the existence of functional thinking. However, one of the important results of the research is that some students had more difficulty in generalizing and representing relationships whose general rule is in the form of $y=mx+n$ than $y=mx$.

Another study on the functional thinking of fifth-grade students was conducted by Akın (2020). Akin (2020) designed a functional thinking intervention consisting of five lesson plans. The problems in the lesson plans were drawn from real-life situations, and the questions they contained encouraged students to discuss the problems. As a result, an increase was observed in the functional thinking skills of the experimental group of students. Akın (2020) also revealed that in the pre-test, while the students were generally inclined to explain the relationships between the variables using recursive patterns, the ability of experimental students to define covariational relationships and state the function rule with words and variables increased. In addition, she noted that students were more successful in defining the $y=2x$ relationship than $y=3x+2$.

On the other hand, Ozturk et al. (2020) examined the effect of an early algebra approach on the functional thinking skills of third-grade students. According to the study results, although the percentage of students answering the problems in the post-test increased in both control and intervention groups, there was a greater increase in the intervention group. In addition, when student strategies were examined, it was observed that the students in the intervention group were found to use more advanced strategies in generalizing and representing functional relationships. Ozturk et al. (2021) also examined the effect of this early algebra approach on students' ability to use variables. According to the findings of the study, the students in the intervention group used letters as variables in different big ideas of algebra, including generalized arithmetic and representing unknown quantities. Also, the intervention group was found to use more algebraic strategies compared to the control group students.

2.4 Game-based Learning

With the advancing and constantly developing technology and internet usage in today's society, the interaction of students with computers, tablets, and phones has increased, and the period devoted to them is also quite high. These tools can be integrated into classroom activities to increase students' interest in these communication tools and increase their motivation for the lesson. The inclusion of game-based activities in mathematics and algebra teaching has a positive effect on students' learning process. With these game-based learning activities, algebra learning can become more engaging, fun, and meaningful so that students can grasp mathematical situations easily. In the literature, there were similar views stating that playing such game activities increased the students' interest in the lesson (e.g., Malone, 1981), they were more motivated to participate in the lesson and they developed more positive attitudes toward math learning (e.g., Ke, 2008; Malone, 1981), they developed deeper comprehension levels, logical thinking and problem-solving (e.g., Kirriemuir & McFarlane, 2004), students' cognitive functions such as critical and strategic thinking enhanced (e.g., Allsop et al., 2013), and they enhanced mathematics performance and encouraged positive math attitudes (e.g., Ke and Grabowski, 2007). In addition, much research supporting this claim has confirmed the teaching effectiveness of playing games (e.g., Dempsey et al., 1996; Rieber, 1996).

Studies indicated that the amount of time spent playing video games for children and adolescents had been consistently increasing. While they spent nearly 7 to 10 hours per week playing video games (Lenhart et al., 2008), this time duration has been increased to approximately 30 to 43 hours per week (Homer et al., 2012). This is supposed that the period continues to rise. The results of the study showed that the average screen time of Turkish adolescents with 3.41 hours per day for the total sample was above the recommended level (Karaca et al., 2011). Allsop et al. (2013) indicated that teachers from Turkey, Italy, and the UK were enthusiastic about teaching using digital games, and the majority believed that digital games were an outstanding teaching instrument for improving a certain curricular aim in a given topic in the elementary classroom. Some teachers stated how games might help students acquire transferrable abilities, including critical thinking, problem-solving, cooperation, and creativity Some views indicate that games enhance students' cognitive functions, such as critical and strategic thinking (Kirriemuir & McFarlane, 2004).

Another critical study dealing with online gaming and early algebra is conducted by Van Den Heuvel-Panhuizen et al. (2013). The researchers investigated how elementary school students used an online game to aid their early algebra problem-solving strategies. For this reason, they designed a dynamic online game for students to explore the relationships between quantities. This game featured a target, bow, and arrows to hit that target, and a board to show the number of hits and misses. For the current study, this game was adapted to make students discover functional relationships. This will be detailed in the methodology part. They examined how a dynamic online game affected students' ability to solve basic algebraic equations. The fourth, fifth, and sixth graders (10 to 12 years old) completed the game as homework at home to solve several early algebraic problems, which involved contextual issues with covarying quantities. When the students were working on the problems online, special software was used to keep track of their progress. A test related to early algebra was given before and after the intervention. A coding schema was developed for problem-solving strategies, and these strategies were divided into two parts answer-focused and relation-focused. They examined the effect of strategies and tested the effect of the level of online working. They distinguished between three levels of online working, namely free playing, primarily looking for answers, and exploring relationships, based on the extent of online involvement and the type of strategy used. The data analysis demonstrated that their online study aided the pupils' early algebra proficiency. Across all grades, there was a dramatic performance improvement. The most significant effect was found in grade 6.

2.5 Summary of the Literature Review

The studies indicated that by systematically improving elementary school students' algebraic thinking through a curricular approach, these students could focus on sophisticated algebraic thinking performance that relies on making generalizations, representing, justifying, and reasoning with mathematical concepts and relationships (Kaput & Blanton, 2011). It was revealed that upper elementary grade students' algebraic comprehension and their algebra preparation for middle school could be improved by comprehensive early algebra interventions (Cai et al., 2011; Warren & Cooper, 2008). It was also shown that lower elementary school students have a capability for algebraic thinking that exceeds what was initially predicted as attainable (Warren & Cooper, 2008). Warren and Cooper (2008) conducted an experiment among 8 years-old students in two sessions, where the first one was about copying patterns and describing them in positional language, and the second one focused on extending their language to describe and predict patterns by re-examining. In their findings, it was stressed that students could learn functional thinking at an early age. Additionally, Ferrara and Sinclair (2016) asserted that having access to sustained experiences, from the beginning of school life, with the conceptual approach to early algebra could help to remediate the students' troubles and underachievement in further algebra subjects. As seen in the national mathematics curriculum, students encounter with algebra learning domain in 6th grade for the first time in Turkey. However, some of the big ideas which involve patterns, the order of the operations, the meaning of the equal sign, and equalities are addressed in the elementary school grade objectives. Findings, according to Blanton et al. (2017), stress that first-grade children can think about variable quantities and notation through functional thinking. A similar idea has been shown to be true in the study of Warren et al. (2006), where results explained that elementary students are able not only to improve their functional thinking but also to communicate it.

The studies about game-based learning indicate that teachers think that digital games are an effective educational tool for achieving a certain curricular goal in a given subject. The effects of games on the growth of transferable abilities, including problem-solving, critical thinking, collaboration, and creativity, were mentioned by several teachers (Allsop et al., 2013). Also, some researchers indicated that playing games enhance students' cognitive functions, such as critical and strategic thinking (Kirriemuir & McFarlane, 2004).

Overall, the literature review provided studies that stress the important component of algebraic reasoning and students' understanding and difficulties in algebraic thinking. It also brought the essence of teaching early algebra to remediate the students' troubles and underachievement in further algebra subjects. The significance of incorporating game-based learning into the classroom was stated. Unfortunately, few studies have been found in the current literature that collectively addresses each of these aspects. Hence, the results of this study aimed to contribute to the literature about investigating fifth-grade students' functional thinking and learning process with game-based learning.

CHAPTER 3

METHODOLOGY

This chapter is devoted to the information about the details of the research design, participants of the study, data collection methods and procedures, instruments, pilot study, analysis of the data, the role of the researcher, reliability and validity issues, and finally, ethics.

The purpose of this study is to investigate the fifth-grade students' development of functional thinking within game-based learning and to explore students' abilities for making generalizations, identifying variables, and representing functional relationships. The following research questions were explored to pursue this goal:

1) How do fifth-grade students' functional thinking processes develop after their engagement in the learning process with game-based learning activities?

a. How do fifth-grade students' generalization process of the functional relationships develop with the implementation of game-based learning activities?

b. How do fifth-grade students' representation process of the functional relationships develop with the implementation of game-based learning activities?

3.1 Research Design

In this study, the qualitative research design was used to expose the fifth-grade students' functional thinking processes in game-based learning and to explore their ability to generalize and represent functional relationships. Qualitative research can be described as "an effort to understand situations in their uniqueness as part of a particular context and the interactions there" (Patton, 1985, p. 1). The meanings individuals attach to circumstances are examined by investigators, who study with qualitative methods in their natural contexts for this research (Denzin & Lincoln, 2005) Qualitative research techniques are intended to assist in exposing how a target audience behaves and thinks about a certain subject.

In this research, the case study, one of the types of qualitative research, was used to investigate the fifth-grade students' functional thinking processes in game-based learning. Yin (2003) defines a case study as “a case study is an empirical inquiry that investigates a phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident” (p. 13). Creswell (2007) states that studying an issue as it is presented through one or more cases inside a constrained system is called case study research. (i.e., a context or situation). The researchers portray it as an investigative technique, a methodology, or an all-encompassing study plan (Denzin & Lincoln, 2005; Merriam, 1998; Yin, 2003). Case study research is a qualitative method in which the researcher collects detailed, in-depth information from various sources (such as observations, interviews, audiovisual material, documents, and reports) to examine one or more bounded systems (cases) over time. The investigator then presents a case description and case-based themes.

According to Creswell (2007), case study categories are differentiated by the scope of the bounded case, such as whether a case covers one individual, many individuals, a group, an entire program, or an activity. They can also be separated by the purpose of the case analysis. With regards to intent, there are three options: the single instrumental case study, the collective or multiple case study, and the intrinsic case study. The one concern or issue is chosen again in a multiple case study, but the inquirer chooses multiple case studies to demonstrate the issue. Frequently, the inquirer picks multiple cases to demonstrate alternative viewpoints on the subject. Merriam (2009) indicates that many case studies entail gathering and evaluating data from several cases. There seem to be two stages of analysis in a multiple case study: within-case analysis and cross-case analysis. Each case is first regarded as a complete case in and of itself for the within-case analysis. Data are obtained so that the researcher may learn as much as possible about the contextual variables that may impact the case. After each instance has been thoroughly examined, cross-case analysis begins. A qualitative, inductive, multi-case research attempts to construct abstractions across cases. Although the exact circumstances of different cases may differ, the researcher strives to provide a general explanation that fits the individual cases (Yin, 2008).

In this thesis study, case study was applied to better investigate four fifth grade students' functional thinking processes deeply. The observations of the whole playing game process for this study included the students identifying the patterns inherent in the game and utilizing the pattern rules to complete the levels in the activity time. The pre-and post-interviews also contributed to understanding the students' ways of thinking or strategies about functional relationships. Thus, when these numerous data sources are considered together, they can provide a holistic explanation for students' improvement in functional thinking processes. A multiple-case study design with a single unit of analysis was used in this study to investigate generalization and representation processes for functional relationships based on the gaming process of four fifth-grade students by examining their worksheets, gaming processes, and responses to pre-and post-tests, and reflections on interview questions. The cases were four fifth-grade students, and the unit of analysis was students' functional thinking processes.

3.2 Participants

In this study, the participants were four fifth-grade students whose generalization and representation processes of functional relationships were investigated with a game-based learning activity. Convenience sampling is a sample selection based on time, money, location, and availability of sites or respondents (Merriam, 2009). When random sampling is difficult, the researchers employ convenient sampling and choose individuals based on location and time availability (Fraenkel et al., 2012; Merriam, 2009). Since the researcher was a teacher in a public school and she taught fifth-grade students, who had not studied algebra before, the participants were selected from the school where the researcher taught. There were two classes the researcher taught at the fifth-grade grade level during the Spring semester of the 2021-2022 academic year when the study was conducted.

Purposive sampling is the most often used nonprobability sampling approach (Merriam, 2009). According to Merriam (2009), purposeful sampling is founded on the concept that the investigator wants to find, comprehend, and acquire insight and hence must select a sample from which the most could be learned. It is the deliberate selection of individuals who supply the most information on a topic when a researcher wants to explore and acquire insight into it. As previously stated, the study tries to choose individuals who give rich data and are more helpful in exploring the topic in-depth rather than generalizing the findings (Creswell, 2012; Patton, 2002). Within this aspect, the sample to be included in the study was selected by considering the pre-test answers given and the ability of the students to express themselves. While selecting students for the main study, the variety of answers they gave in the pre-test and their levels were considered, and these levels were coded according to Stephens et al. (2017).

In the pre-test, there were 33 students. Nineteen of them were girls, and 14 of them were boys. For the main study, four participants, who were 2 girls and 2 boys, were selected from this group. In explaining functional relationships, while Yavuz's focus was on covariational relationships, the most common one in Funda's answers was recursive patterns. Harun's answers included various levels such as RPG, CR, and FCW. In addition to these, Harun's answers in the pre-test included symbolic representation forms. He tended to use symbols when expressing the function rule. Zeynep, on the other hand, explained functional relationships by giving more upper-level answers such as FCW and FEW in the pre-test.

Zeynep was a student above the average in terms of academic success, but there were also students who were academically higher than her in her class. She had a shy and calm disposition, so she was a little hesitant to have a say in the lesson. However, when she was asked a question, she was open to answering and finding solutions in different ways.

Yavuz was a student just below the average in his class in terms of academic success. He had a calm disposition. He usually tried to be active in the lessons; he could express his thoughts and answers when given the right to speak.

Harun was an average student in his class in terms of academic success. He was usually active in lessons and talkative. When there were parts that he did not understand, he was prone to ask and learn.

Funda was an academically low student in her class. She had a calm and quiet disposition and did not usually try to take a voice in the lessons. But when asked a question, she tried to explain herself and her ideas, even if she was a little hesitant.

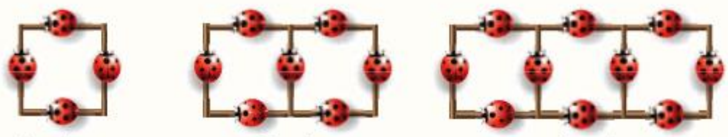
3.2.1 The Context of the Study

This study was implemented in a middle school in Mardin. The school had 21 teachers, 13 classrooms, and around 250 students. There were smart boards and the internet in the classrooms, and the lessons were usually taught interactively from the smart board. Fifth-grade students took 5 hours of mathematics lessons per week as a compulsory mathematics course throughout the year. Apart from this, students did not have any elective mathematics courses.

The students were taught the subject of patterns in the first semester. Table 3.1 presents the objective and lesson duration of the pattern subject. While this subject was being studied, similar methods and teaching techniques were applied to both classes based on the textbook and resources published by the Ministry of National Education. One of the examples solved from the textbook on patterns is shown in Figure 3.1. In this question, students were given a pattern of shapes and asked questions about the number of ladybugs and the number of squares for different steps.

Table 3.1 Date and Objectives of the Pattern Subject

Date and Duration	Objectives
September 20th -26th, 2021 (5 hours)	M.5.1.1.3. Students form the desired steps of the given number and shape patterns. a) Restricted only to patterns with a constant difference between steps. b) Examples of our historical and cultural artifacts (architectural structures, carpet ornaments, rugs, etc.) are given to the figure patterns.



1st Step **2nd Step** **3rd Step**

The above pattern grows by adding new squares and ladybugs after step one. Accordingly, let's answer the following steps.

a) How many squares and ladybugs are in step 4 of the pattern?
b) In which step of the pattern are there 22 ladybugs?
c) Which step of the pattern contains 6 squares?
d) In which step of the pattern are there 28 ladybugs?
e) How much more is the number of ladybugs in step 5 of the pattern than the number of squares?

Figure 3.1 A Textbook Question Used in the Lesson
Note. One of the solved examples about patterns in the textbook. From Fifth Grade Mathematics Textbook, 2019, p. 21, Tuna Publishing

In teaching the patterns, the lesson started with a small introductory activity or a short problem-solving activity to remind students of their previous knowledge. Then, the students were briefly informed about what they would do in that lesson. Lecturing and activity parts were made from the textbook together with the students. In addition, interactive content, videos, or images related to the subject were shown from the smart board. Figure 3.2 is an image from the video that was shown to the students in the lesson on the subject of patterns.

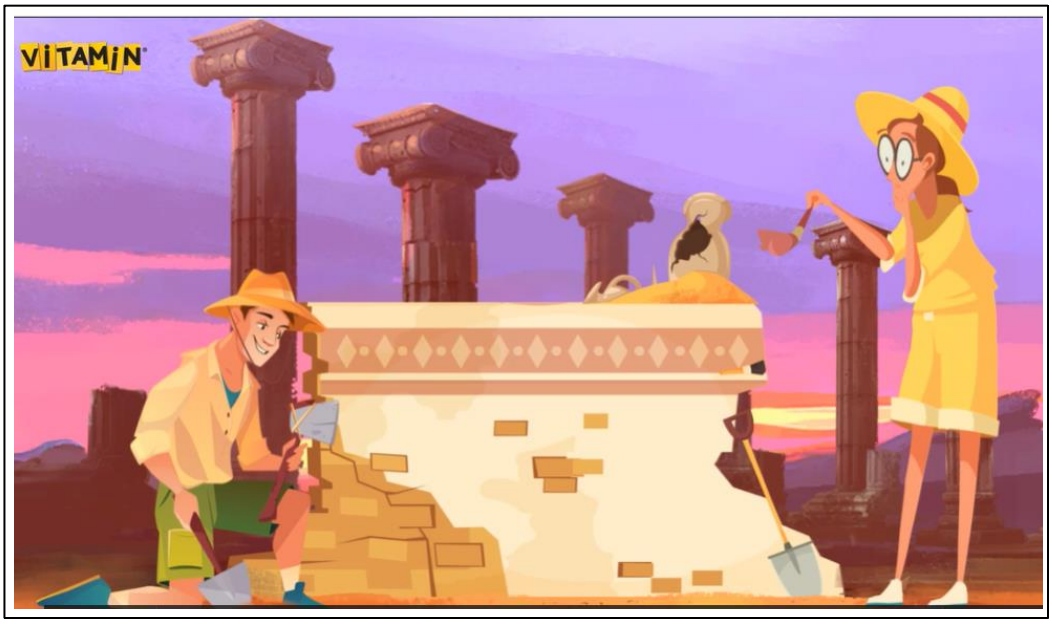


Figure 3.2. Part from the Video related to the Patterns from the Smart Board

Note. An animated video with information showing how the restoration relates to the pattern. From restoration, by Education Information Network, n.d.

After the subject was covered, reinforcement tests were distributed and problems were solved on the smart board. While the subject of patterns was being studied, the exercise questions solved with the students on the smart board are given in Figure 3.3.

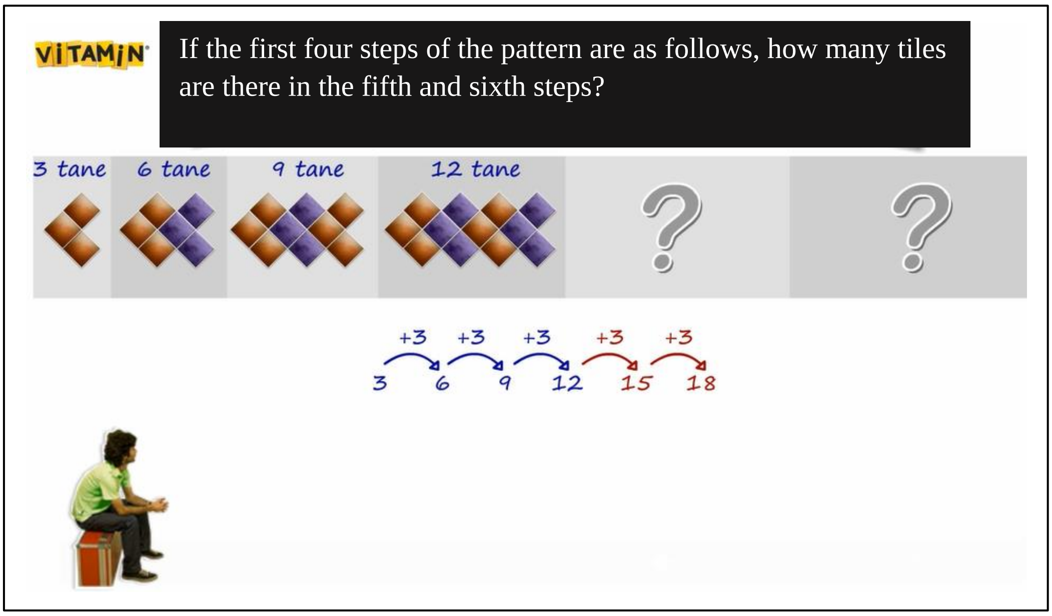


Figure 3.3. An Example of the Exercise Questions Solved with the Students on the Smart Board

Note. A video explaining the pattern rule using figure and number patterns and how the pattern can continue depending on the rule. From Pattern Definition, by Education Information Network, n.d.

3.2.2 Functional Thinking Game

Using the work of Van den Heuvel-Panhuizen et al. (2013), a game containing functional relationships was designed by the researcher, and an interview protocol was developed to be applied to the students afterward. The game was designed using the Scratch application, which was a programming language with different interfaces. Using the interface of this application, tools such as dartboard, background, arrows, and scoring table was added, and different levels were designed. For this point, the researcher also received help from an expert in computer and instructional technology education for game development. The game aimed to provide different sub-acquisitions of functional thinking and different functional relationships, such as recursive patterns, covariational thinking, and one-to-one correspondence, by exploring them with game-based learning. The first section included values for the y=mx function and the second for the y=mx+n function. It aimed to have the player create a pattern with different numbers and establish a relationship between the number of shots, total points, missed shots, and hits. At the same time, it aimed to create a learning infrastructure for algebra subjects in later classes, such as finding patterns, creating variables, and establishing equations by trying to understand the rules of the game and developing different strategies.

The game consisted of two parts. Each part contained a different learning outcome, and various sub-levels from easy to difficult. There was a target board in the game, arrows to be shot at the target, and a score table where the shots and scores were recorded (see Figure 3.4). It was recommended that setting specific goals and rules for the game increased players' attention, efforts, and motivation and also it affected students' learning process positively (Garris et al., 2017; Locke et al., 1990; Mayer et al., 2002; Swaak et al., 1998). In line with these, the player was given certain score targets in each section and was expected to complete this score with the minimum number of shots in different conditions. They had to reach the targeted points with the minimum number of shots to pass the levels.

Figure 3.4 Screen from the Game
https://scratch.mit.edu/projects/546787831/fullscreen/

This information was presented to the students before starting each level in the game. An example is given in Figure 3.5. In the Level 1.3 of the game, before starting, the informative text on the screen says, 'Each accurate shot gives 3 points, and the target score is 30. When you reach 30 points, you can move on to the next level.' After reading this text, the student starts shooting. The scoreboard in the lower left corner of the screen is updated after each shot of the student. By following this table, the student can discover the relationships between the number of shots and points in the game. Students played the game individually on the computer under the supervision of a researcher in a separate room. At this point, the role of the researcher was to observe the student and try to understand their processes of exploring relationships by asking them to think aloud. The researcher did not intervene in the game process; she just asked the students to think aloud while playing the game and share the steps with her. This level of the game was prepared for the y=3x function, and the relationships between the quantities in the game were designed to explore this function.

Figure 3.5 Screen from Level 1.3 in the Game

In Figure 3.6, the visual of the Level 2.2 of the game was presented. At this level, the game starts with 1 point, and the points obtained for accurate shots are added to it. Each accurate shot earns 4 points, and the target score is 25. This level of the game was prepared for the y=4x+1 function, and the relationships between the values on the scoreboard were designed to explore this function. While playing this game, students must follow the scoreboard to reach the target score. By looking at the values there, the students can see the number of hits and the total score and think about how many shots they need to make and how many more points they need to earn to reach the target score to pass the section. In this way, they can explore functional relationships by concentrating on the relationships between these quantities.

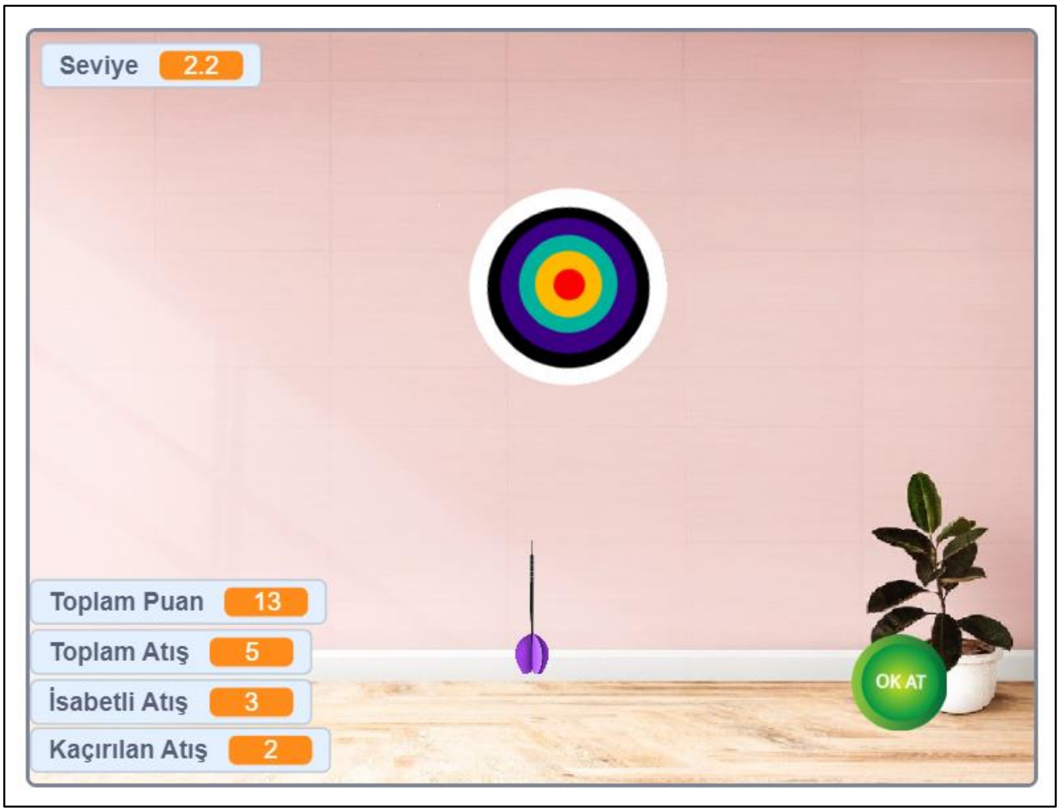


Figure 3.6 Screen from Level 2.2 in the Game

3.3 Data Collection Procedures and Data Sources

The cases to be included in the study was selected by considering majorly the answers given in the pre-test and the ability of the students to express themselves. In this direction, the answers of the students in the pre-test were coded according to their generalization levels and representation forms. Considering these coding, students with different generalization levels and various forms of representation were selected. Afterward, pre-interviews were held to understand the solutions and ways of thinking about the answers they gave in the pre-test in more detail. A game-based learning activity was carried out with this sample as a second stage. They played the game individually on the computer under the supervision of a researcher and in two parts in an empty room. Apart from this, no extra time was given to the students for homework or to play the game separately. A game interview was held with these students after this activity. In this interview, six different problems related to functional thinking were asked in parallel with the parts in the game, and the answers to these questions were recorded, and the students' worksheets were collected. As a final step, a post-test was applied to examine the development of students' functional thinking processes. After this post-test, post-interviews were held with the students to understand and analyze their answers, strategies, and ways of thinking deeper.

The data was gathered during the Spring semester of the 2021-2022 school year (see Table 3.2). The data collection procedure began once permissions were acquired from the University Human Subjects Ethics Committee (see Appendix C) and the Ministry of National Education (see Appendix D). The data collection process began when the written consent documents were received from the parents. The data gathering procedure schedule is shown in Table 3.2.

Table 3.2 Time Schedule of the Study

Date	Administration
2020-2021 Academic Year	Conducting the pilot study 1 (Game and game interview)
	Conducting the pilot study 2 (Game, pre-test, post-test, and interviews)
	Conducting the pilot study 3 (pre-test, post-test, and interviews)
April 14th	Pre-test
May 2nd –6th	Pre-interviews
May 9th -13th	Game + Interview
June 13th	Post-test
June 14th -17th	Post-interviews

3.4 Instruments

In this study, the Functional Thinking Test (FTT) and semi-structured interviews were used as data tools. Each instrument that was used in the study will be explained in detail in this part.

3.4.1 Functional Thinking Test

The current study sought to explore students' existing functional thinking and improve their generalization and representation processes of functional relationships. Within the scope of this research, the Functional Thinking Test (FTT) was designed to explore different sub-acquisitions of functional thinking and different functional relationships, such as recursive patterns, covariational thinking, and one-to-one correspondence thinking. The FTT was created to examine students' functional thinking processes. Hence the main problems involved $y=mx$ and $y=mx+b$ equations (see Appendix A). Students were expected to identify data, organize the data in a table, describe patterns in this table, and define the rule of the relationship between two quantities in variables or symbols and words using these questions.

Item 1, Brady task, was adapted from Stephens et al. (2017). In the Brady task, there were two main problems with the able seating arrangement: the first one was related to the $y=mx$ functional relationship and had five sub-questions, and the second one was related to the $y=mx+n$ functional relationship with five sub-questions. These problems were combined and gathered under one main problem with nine sub-questions.

Item 2 required students to define the $y=3x+4$ functional relationship, adapted from Wilkie (2015). The content of this problem has not been changed, but its sub-questions have been revised to be parallel to the first problem. As a result, it has become the main second problem with five sub-questions.

The Functional Thinking Test was created in accordance with the instructional objectives of the Grades 1-8 National Mathematics Curriculum (MoNE, 2018), which are shown in Table 3.3.

Table 3.3 Objectives addressing functional thinking in Grades 1-8 (MoNE, 2018)

Objectives	Items in the tests
M.3.1.1.7 Students expand and generate the number of patterns that have a constant difference.	1a, 1b, 1f, 1g, 2a 2b
M.4.1.1.6 Students create several patterns that increase or decrease according to a certain rule and explain the rule of these patterns.	1a, 1b, 1f, 1g, 2a, 2b
M.5.1.1.3 Students find the required steps of the given number and figure patterns. M.7.2.1.3 Students express the rule of the number patterns using letters and find the asked term of the pattern when the rule was expressed by letters.	1d, 1i, 2d,
M.7.2.2.2 Students identify linear equations with one unknown and construct a linear equation with one unknown corresponding to the given real-life situations.	1c, 1d, 1h, 1i, 2c, 2d
M.8.2.2.5 Students formulate equations, tables, and graphs for real-life situations involving linear relationships and interpret them.	1c, 1d, 1h, 1i, 2c, 2d

Table 3.3 (continued)

Objectives	Items in the tests
M.7.2.2.3 Students solve equations with unknown.	1e, 2e
M.7.2.2.4 Students solve the problems that require constructing linear equations with one unknown.	1e, 2e
M.8.2.2.1 Students solve the problems that require constructing linear equations with one unknown.	1e, 2e
M.8.2.2.3 Students express how one of the variables changes in relation to the other using a table and an equation when there is a linear relationship between the variables.	1b, 2b

An expert in the early algebra field of mathematics education examined the test questions for content validity. A pilot study was also carried out. The test was changed in response to feedback. These changes will be detailed next.

In the beginning, there were four questions, and each of these questions had sub-questions. It was thought that it would take a long time to apply this test, and a pilot study was conducted to try both the application time and the questions. As a result, it was reduced to two questions. Expert opinion was obtained for the test questions, and some changes were made in the structure of the questions accordingly. A few changes were made to the wording of the questions. Besides this, in item 1b, 'Examine the relationship between the number of tables and the number of people. How can you express these relationships in different ways?' was asked. The expert thought this might not be clear enough, and accordingly, this question was divided into two questions "Define the patterns in the table." and "Write the rule in words that explains the relationship between the number of tables and the number of people.” In addition, in item 1d, the question was expressed “If Burak adds 2 more people to the sides of the table, how will this affect the rule you wrote in option c?” It was formatted as “If Burak adds 2 more people to the sides of the table, fill in the table below considering how many people can sit at the tables in the new situation.” Parallel to these, the second problem and sub-questions were also arranged. After these changes, the final version of the test was edited. The test consisted of two main problems, the first of which had nine sub-questions and the second of which had five sub-questions. Tables 3.4 and 3.5 show the goals of each question on the functional thinking test.

In item 1, students were given a contextual problem regarding the table seating arrangement on the birthday, and they were expected to identify, represent, and generalize the functional relationships using variables, symbols, and words. Table 3.4 lists item 1, its sub-questions, and the aims addressed.

Table 3.4 Item 1 of The Functional Thinking Test and Aims Addressed

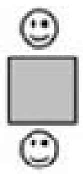
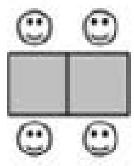
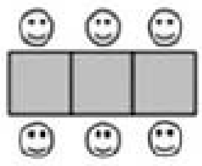
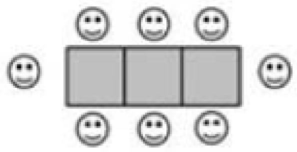
Item 1	Aims
Burak invites his friends to his birthday party. He wants to make sure there is a seat for each of her friends around the square tables. He can seat 2 people at one desk in the following way:  If he joins another desk to the first one, he can seat 4 people:  If he joins another desk to the second one, he can seat 6 people: 	

Table 3.4 (continued)

<table>
<tr><th>Item 1</th><th>Aims</th></tr>
<tr><td>a) Fill in the table by thinking about how many people Burak can seat at different numbers of desks?
<table>
<tr><th>Masa Sayısı</th><th>Kişi Sayısı</th></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td>5</td><td></td></tr>
<tr><td>…</td><td></td></tr>
<tr><td></td><td></td></tr>
</table></td><td>Generating the data and organizing it in a table</td></tr>
<tr><td>b) Explain the patterns in the Table.</td><td>Identifying patterns</td></tr>
<tr><td>c) Use words to write the rule that describes this relationship between the number of desks and the number of people.</td><td>Identifying the function rule in words</td></tr>
<tr><td>d) Express the rule showing this relationship using symbols or letters such as boxes, stars, circles and find the value of these symbols or letters.</td><td>Identifying the function rule in symbols or variables</td></tr>
<tr><td>e) If Burak has 100 desks, how many people can he seat? Show how you got your answer.</td><td>Using the function rule to predict far function values</td></tr>
</table>

Table 3.4 (continued)

Item 1	Aims
f) Burak figured out he could seat more people if two people sat on the ends of the row of desks. For example, if Brady had 3 desks, could seat 8 people.	Generating the data and organizing it in a table
	
If two people sat on the ends of the row of desks, fill in the table by thinking about how many people Burak can seat at different numbers of desks.	
g) Explain the patterns in the Table.	Identifying patterns
h) Use words to write the rule that describes this relationship between the number of desks and the number of people.	Identifying the function rule in words
i) Express the rule showing this relationship using symbols or letters such as boxes, stars, circles and find the value of these symbols or letters.	Identifying the function rule in symbols or variables

Masa Sayısı	Kişi Sayısı
1	
2	
3	
4	
5	
6	
7	

Item 2, adapted from Wilkie (2015), required students to define the y=3x+4 functional relationship. Following the pilot study, certain modifications were made to the item. Section 3.5 goes into further depth about these modifications. Students were given a planting context problem and asked to identify, represent and generalize the functional relationship using variables, symbols, and words. Table 3.5 lists item 2, its sub-questions, and the aims addressed.

Table 3.5 Item 2 of The Functional Thinking Test and Aims Addressed

Item 2	Aims
Ceren likes flowers and plants, so he planted a sapling in her garden. The plant on first day has 4 leaves and each day the plant has 3 new leaves.	
a) Form and fill a table to show the relation between the number of days and number of leaves.	Generating the data and organizing it in a table
b) Explain the patterns on the table.	Identifying patterns
c) Use words to explain the relation between the number of days and number of leaves.	Identifying the function rule in words
d) Express the rule showing this relationship using symbols or letters such as boxes, stars, circles and find the value of these symbols or letters.	Identifying the function rule in symbols or variables
e) At the end of 100^{th} days, how many leaves does the plant have? Show how you got your answer.	Using the function rule to predict far function values

3.4.2 Interview Protocols

Heuvel-Panhuize et al. (2013) stated that accompanying with game playing process working on the problems that enable the discovery of relationships between the quantities in the game allows the learning effects to emerge. Based on this, I have integrated game-parallel problems into the interview protocol. After playing the game, the game interview was conducted with the participants. The test includes six main problems related to game levels: three of these problems for the first section of the game and three for the second section. These items and their aims addressed were presented in Tables 3.6 and 3.7.

Table 3.6 Interview Protocol Items for the First Part of The Game and Aims Addressed

Items	Aims
a) A player who gains 7 points for each hit reaches 56 points at minimum how many hits?	
Show the relationship between the total score and the hits in the table.	Generating the data and organizing it in a table
Explain the patterns in the table.	Identifying patterns
What is the rule that explains the relationship between the number of hits and the total score?	Identifying the function rule in words
Express the rule defining this relationship using letters or symbols such as boxes, stars, and circles.	Identifying the function rule in symbols or variables

Table 3.6 (continued)

Items	Aims
b) If a player who gains 5 points for each hit and misses 3 shots, in how many shots will he reach the target of 30 points?	Using the function rule to predict far function values
Show the relationship between the total score and the hits in the table.	Generating the data and organizing it in a table
Explain the patterns in the table.	Identifying patterns
What is the rule that explains the relationship between the number of hits and the total score?	Identifying the function rule in words
Express the rule defining this relationship using letters or symbols such as boxes, stars, and circles.	Identifying the function rule in symbols or variables
c) The player missed four out of 10 shots. If he scores 42 points in total, how many points does she gains for each hit?	
Show the relationship between the total score and the hits in the table.	Generating the data and organizing it in a table
Explain the patterns in the table.	Identifying patterns
What is the rule that explains the relationship between the number of hits and the total score?	Identifying the function rule in words
Express the rule defining this relationship using letters or symbols such as boxes, stars, and circles.	Identifying the function rule in symbols or variables

Table 3.7 Interview Protocol Items for the Second Part of The Game and Aims Addressed

Items	Aims
a) A player who starts the game with 4 points and gains 7 points for each hit will reach 60 points with a minimum of how many shots?	
Show the relationship between the total score and the hits in the table.	Generating the data and organizing it in a table
Explain the patterns in the table.	Identifying patterns
What is the rule that explains the relationship between the number of hits and the total score?	Identifying the function rule in words
Express the rule defining this relationship using letters or symbols such as boxes, stars, and circles.	Identifying the function rule in symbols or variables
b) A player who started the game with 4 points and gained 5 points for each hit and missed 3 shots. With how many shots will he reach the target of 44 points?	
Show the relationship between the total score and the hits in the table.	Generating the data and organizing it in a table

Table 3.7 (continued)

Items	Aims
Explain the patterns in the table.	Identifying patterns
What is the rule that explains the relationship between the number of hits and the total score?	Identifying the function rule in words
Express the rule defining this relationship using letters or symbols such as boxes, stars, and circles.	Identifying the function rule in symbols or variables
c) A player who started the game with 7 points missed five of 12 shots. If he scores 70 points in total, how many points is each hit?	
Show the relationship between the total score and the hits in the table.	Generating the data and organizing it in a table
Explain the patterns in the table.	Identifying patterns
What is the rule that explains the relationship between the number of hits and the total score?	Identifying the function rule in words
Express the rule defining this relationship using letters or symbols such as boxes, stars, and circles.	Identifying the function rule in symbols or variables

3.5 Pilot Study

Using the work of Van den Heuvel-Panhuizen et al. (2013), the researcher designed a game containing functional relationships. In the pilot study for the game, some problems were encountered and it was concluded that revisions were needed. In this process, the researcher met with an expert in computer and instructional technology education and received help in the game development. In the beginning, only a general explanation about the game and the levels was given in the game introduction. But later, it was thought that it would be more appropriate to give short information before starting the game for each level, and accordingly, short information notes were added to each section. In addition, changes have been made, such as the dartboard moving in different ways or speeding up and shrinking to make the game more exciting and flow better. In light of these, a new game version was created.

The pilot study of the test lasted approximately two lesson hours. Twenty students participated in this test. This test included open-ended real-life problems; 20 sub-questions in total under four main questions. At the end of the pilot study, some changes were made to the questions in the test. Some changes were made to keep the test time shorter and clarify the wording of some questions.

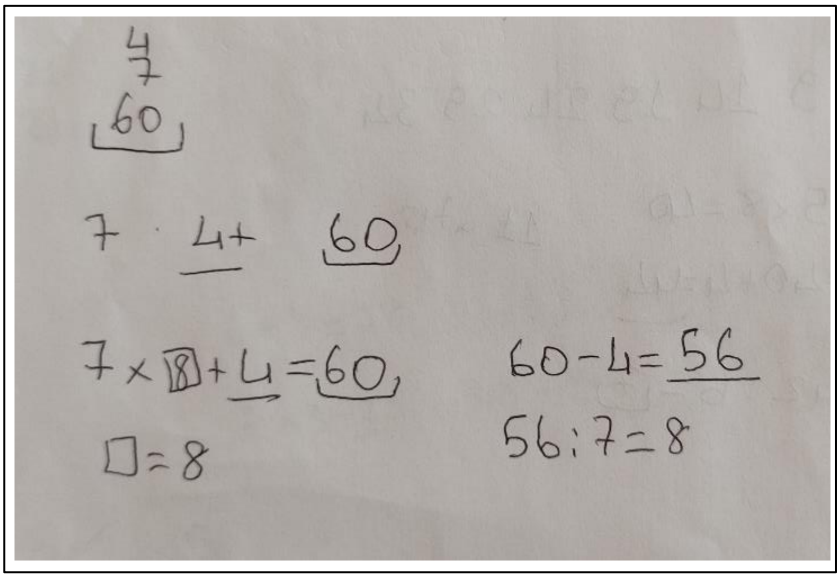


Figure 3.7 Student answer from the Pilot Study

In addition to these, in the pilot study of the game and the game interview, the students were found to use symbols such as box, star, and circle to write the function rule in addition to the use of letter variables and words while expressing the relationships between quantities (see Figure 3.7). In Figure 3.7, the student's response to the item of the test for the second part of the game was shown. In this question, students were asked, "A player, who starts the game with 4 points and gains 7 points for each hit, will reach 60 points with a minimum of how many shots?" and were expected to express the relationships in different ways. To describe this relationship, students used a box symbol for unknown quantities besides some arithmetic processes. This meant that students tended to use symbols as variables. Therefore, in the instrument, the question of "use variables (letters) to write the rule that describes this relationship" was changed to "Write the rule showing the relationship between the number of table and number of people (number of days and number of leaves for the second problem) by using symbols or letters such as boxes, stars, circles." In parallel to this, the coding framework from the work of Stephens et al. (2017) was adapted. This adaptation will be mentioned in the next chapter.

3.6 Data analysis

Data were analyzed by qualitative methods. In the qualitative part of the data analysis, students' answers were assessed through correctness and strategies. Therefore, a coding guide was created based on Stephens et al. (2017) (see Figure 3.8). The interviews in this study, which were carried out in 3 parts, were transcribed separately. These transcripts were examined in detail, important parts were highlighted, and analysis tables were created and compiled. These compiled parts were coded according to the coding guide, which included the levels of sophistication describing students' generalization and representation processes of functional thinking. This coding guide was adapted in line with the pilot study.

Stephens et al. (2017) determined the developmental level of students' thinking in generalizing and representing functional relationships during a teaching experiment. They defined the relationships between quantities in three categories recursive, correlational, and correspondence thinking. As seen in Figure 2.2 in the literature part, these categories were divided into sub-levels according to the generalization and representation ways used by the students while answering the questions. They located the word categories at a higher level since students were able to define the function rule in variables instead of words more easily. Instead of going to the next level right away, students could progress by skipping these levels. While adapting, the symbols were added to the functional emergent in variables, functional condensed in variables, and functional particular codes (see Table 3.8). The reason for this was that in the pilot study, the students used various symbols instead of letters as variables because they were unfamiliar with them. The students' responses were coded item by item. If there were more than one response, the most sophisticated response was coded to create the data tables.

Table 3.8 Generalization Levels of Functional Thinking

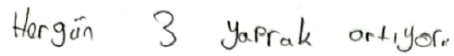
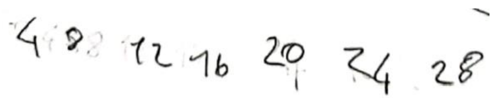
Levels of Sophistication	Description of Levels	Example
No response	The student does not provide a response.	
L0: Restatement	The student restates the given information.	*Two people can sit at a table* 
L1: Recursive pattern-particular	The student identifies a recursive pattern in either variable by referring to particular numbers	*It goes 2, 4, 6, 8, …* 

Table 3.8 (continued)

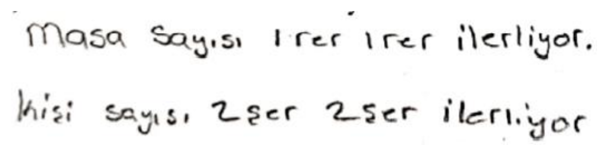
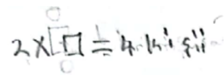
Levels of Sophistication	Description of Levels	Example
L2: Recursive pattern-general	The student identifies a correct recursive pattern in either variable.	*The number of people goes up by 2 each time.* 
L3: Covariational relationship	The student identifies a correct covariational relationship. The two variables are coordinated rather than mentioned separately.	
L4:Single Instantiation	The student writes an expression or equation with numbers and/or unknowns that provides one instantiation of the function rule but does not generally relate the two variables. *This can also be done by spelling out the one instantiation of the function rule*	*2x2 = 4* 

Table 3.8 (continued)

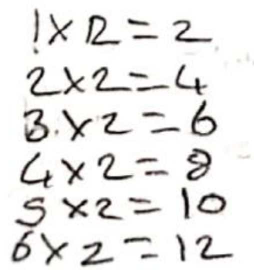
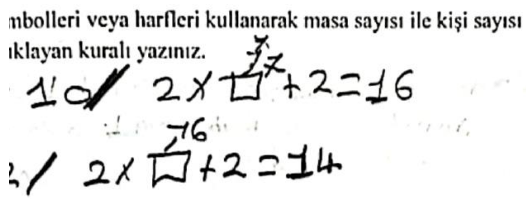
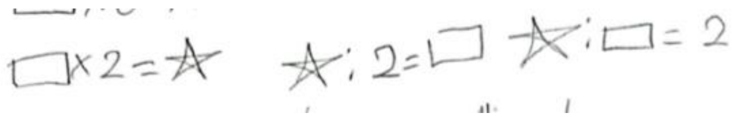
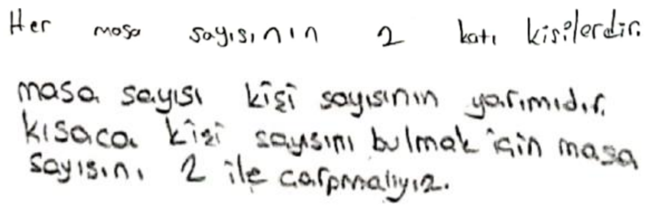
Levels of Sophistication	Description of Levels	Example
L5:Functional-particular/Functional Particular Symbols	The student identifies a functional relationship using particular numbers or symbols but does not make a general statement relating to the variables.	*1x2 = 2, 2x2 = 4, 3x2 = 6, 4x2 = 8, …*  
L6: Functional-basic	The student identifies a general relationship between the two variables but does not identify the transformation between	*Times 2*

Table 3.8 (continued)

Levels of Sophistication	Description of Levels	Example
L7: Functional-emergent in variables/symbols	The student identifies an incomplete function rule using variables or symbols, often describing a transformation on one variable but not explicitly relating it to the other. A student might set the expression equal to a specific number of the same variable rather than a new variable.	*dx2* 
L8: Functional-emergent in words	The student identifies an incomplete function rule in words, often describing a transformation on one variable but not explicitly relating it to the other or not clearly identifying one of the variables.	*You multiply the desks by 2.*

Table 3.8 (continued)

Levels of Sophistication	Description of Levels	Example
L9: Functional-condensed in Variables/symbols	The student identifies a function rule using variables in an equation that describes a generalized relationship between the two variables or symbols, including the transformation of one that would produce the second.	*p = dx2* 
L10: Functional-condensed in words	The student identifies a function rule in words that describes a generalized relationship between the two variables, including the transformation of one that would produce the second.	*If you multiply the number of desks by 2, you get the number of people who can sit.* 

In addition, a coding scheme was assigned for their answers to 1e and 2e sub-questions. In items 1e and 2e, “If Burak has 100 desks, how many people can he seat? Show how you got your answer.” and "At the end of 100th days, how many leaves does the plant have? Show how you got your answer." respectively, participants were supposed to use the function rule to obtain the value for the dependent variable given the value for the independent variable. In Stephens et al. (2017) levels of sophistication, there were no strategy codes for these items. At this point, the codes required for the analysis of these items are provided by Blanton et al. (2015) 's study was developed by utilizing the function rule strategy codes. As a result, students' responses were analyzed, and several strategies were found. Table 3.9 provides the coding scheme for items 1e and 2e. In addition to these, the Function Rule (FR) code was constituted to use the function rule to obtain the value for the dependent variable given the value for the independent variable. When the student finds the result using an incorrect function rule, this was coded as Incorrect Function Rule (IFR). And finally, if the result was obtained from the count of some elements in a pictorial representation, this was coded as Counting (C). Some students answered questions in ways that did not correlate to any level and were therefore irrelevant to the study, or the response was indistinguishable. These responses were coded as "Other (O)" (see an example in Table 3.9). Furthermore, the "Answer Only (AO)" code was used when students provided only an answer without displaying their work. The "No Response (NR)" code was utilized when students left the item blank.

Table 3.9 Coding scheme for Items 1e and 2e in the Test

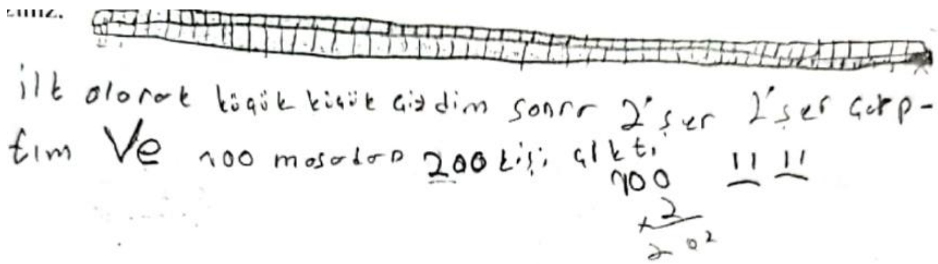
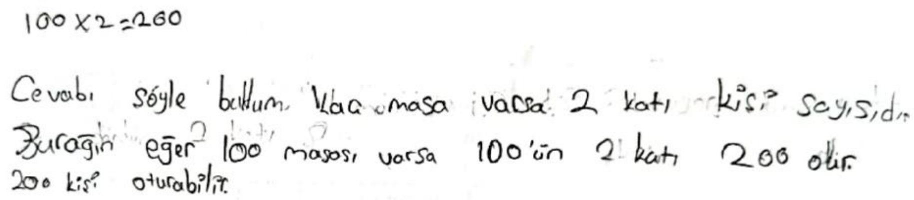
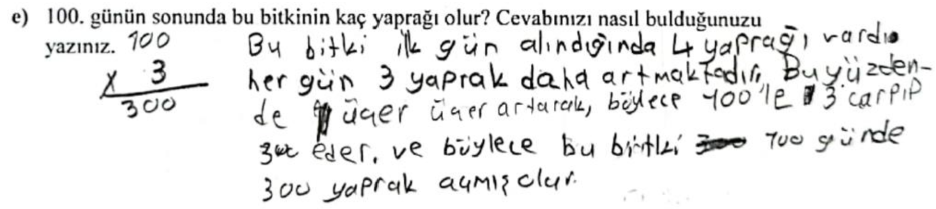
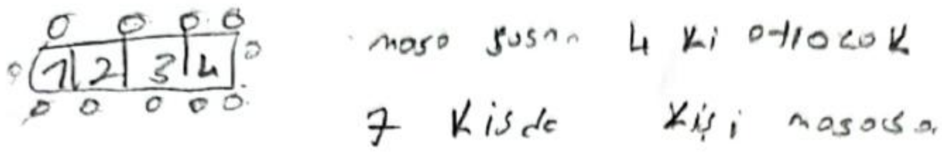
Strategy Code	Description	Example
No Response (NR)	The student does not give an answer.	
Answer Only (AO)	Answers were obtained with no specification of the procedure followed.	200 kişi
Counting (C)	The result was obtained from the count of some elements in a pictorial representation.	

Table 3.9 (continued)

Strategy Code	Description	Example
Function Rule (FR)	The student finds the result by using the function rule.	
Incorrect Function Rule (IFR)	The student finds the result by using an incorrect function rule.	
Other (O)	The student produces a strategy that differs from the above or the strategy is not discernible.	

The works of Pinto and Canadas (2021) and Urena et al. (2022) were used in refining the codes for the representations of functional thinking. Pinto and Canadas (2021) investigated how third-year students relate to and represent the relationships between variables while working on functional problems involving a function of type y=mx+n that they have not studied before. The categories used in the research to analyze the students' answers were presented in Table 2.2 in the second chapter. They created seven sub-categories to analyze the forms students used to represent functional relationships. These categories include natural language, manipulative, pictorial, numerical, algebraic notation, tabular, and multiple representations. Multiple representations are all the possible combinations of more than one of the types of representations.

Urena et al. (2022) examined the strategies and representations sixth-grade primary school students used to generalize functional relationships. Students were expected to answer the questionnaire including functions, divisibility, number operations, measurement, and constructions that point to various linear functional relationships in various situations. Students are encouraged to draw pictorial representation, organize it inductively, and come up with various solutions throughout the problem-solving stage. When the students indicated a general rule connecting the variables by a recognized regularity, they were considered to have expressed generalization. They discussed the many classification schemes used to categorize students' representations of generalization, including verbal, symbolic, and multiple.

Based on these studies, coding categories were created for the forms of representation used by students to generalize functional relationships. A graphical representation form has been added to these categories Table 3.10 presents representation codes, their descriptions, and examples.

Table 3.10 Representation Forms of Functional Thinking

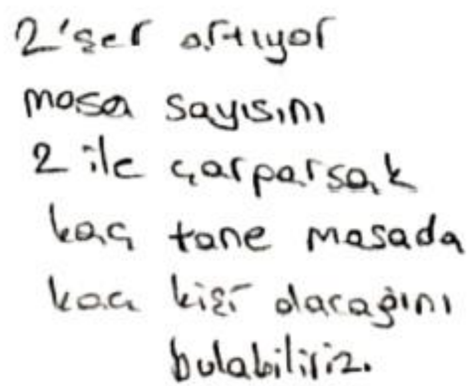
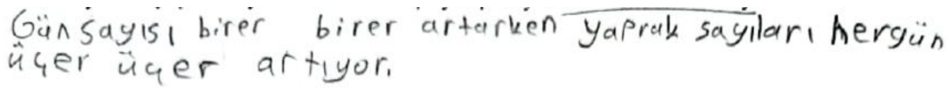
Representations of Generalization	Description	Example
NRep: Not represent the generalizations)	The student does not represent the generalization.	
V: Verbal	The detected regularity is expressed through natural language.	 

Table 3.10 (continued)

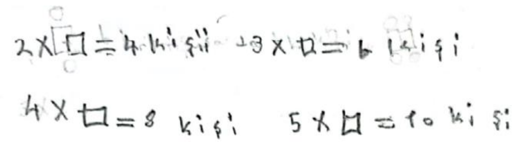
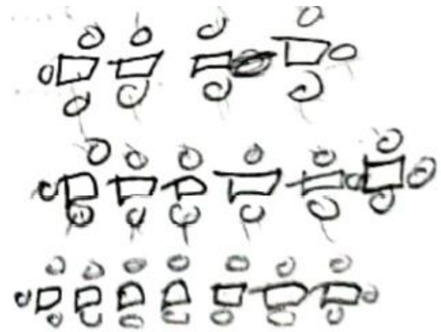
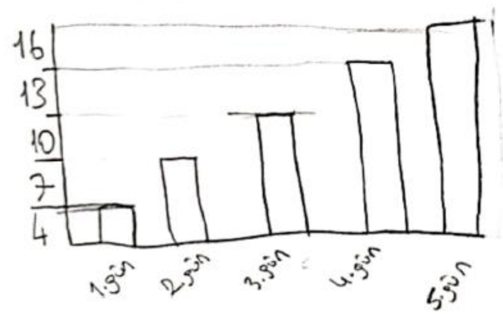
Representations of Generalization	Description	Example
S: Symbolic/Variable	The detected regularity is expressed using algebraic symbolism or symbols such as box, star, and circle.	
P: Pictorial/Visual	The students' solutions are represented visually by translating mathematical problems into the picture. In addition, visual representation involves creating and forming models that can represent mathematical information, so that anyone can understand it easily.	
G: Graphical	It will refer to a graphic representation when the function appears on a coordinate system.	

Table 3.10 (continued)

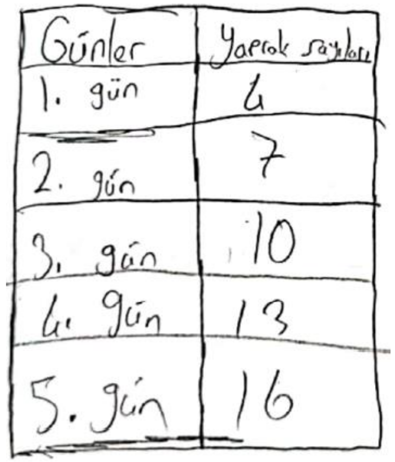
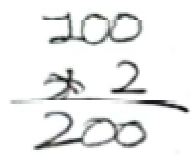
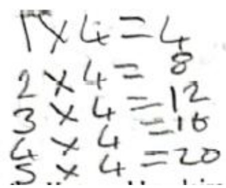
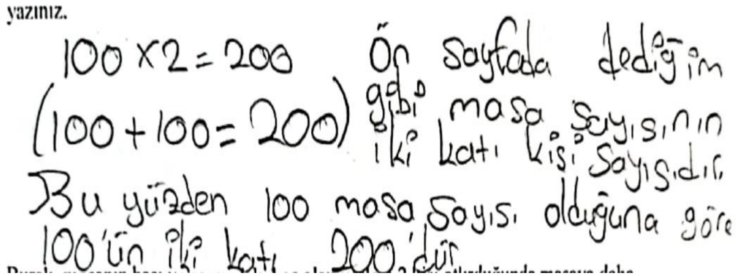
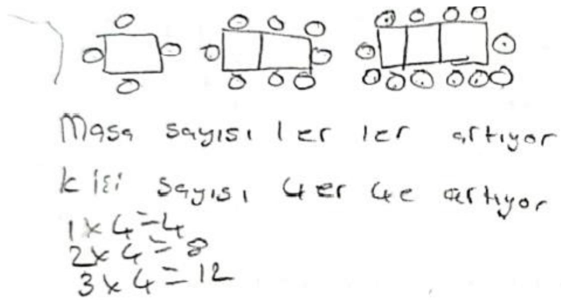
Representations of Generalization	Description	Example
T: Tabular	The tabular representation will refer to a linear function when it is represented by a table or set of ordered pairs.	
N: Numeric	It will refer to numeric representation when solutions were written in numeric forms or when making some mathematical operations with numbers	 

Table 3.10 (continued)

Representations of Generalization	Description	Example
Mltp: Multiple	The detected regularity is expressed using a combination of at least two representations.	 

In light of all this, correctness, generalization levels, and representation forms codes were created to analyze student responses. Some abbreviations were created since these codes were mentioned continuously in the following parts of the study. These coding categories and their abbreviated versions are presented in Table 3.11.

Table 3.11 Coding Table

Correctness	*Levels*	*Representations Of Generalization*
1: Correct	L0: No response or restatement	NRep: Not represent the generalizations
0: Incorrect	L1: Recursive Pattern-Particular (RPP)	V: Verbal
NR: No Response	L2: Recursive Pattern-General (RPG)	S/Var: Symbolic/Variable
	L3: Covariation Relationship (CR)	P: Pictorial/Visual
	L4: Single Instantiation (SI)	Mltp: Multiple
	L5: Functional-Particular (FP) / Functional Particular in Symbols (FPS)	G: Graphical
	L6: Functional-Basic (FB)	T: Tabular
	L7: Functional-Emergent in Variables (FEV) or Symbols (FES)	N: Numeric
	L8: Functional-Emergent in Words (FEW)	
	L9: Functional-Condensed in Variables (FCV) or Symbols (FCS)	
	L10: Functional-Condensed in Words (FC-W)	

3.7 The Motivation and the Role of the Researcher

Since I like to play games in my spare time and have observed that learning by having fun in the activities I do with the students has a positive effect, I thought about integrating games into my work on functional relationships. I had seen the work of my friends who were interested in computers on coding, and I was interested in this field. I did research on this and started watching instructional videos about a few coding and content preparation programs I had heard about before. I decided to develop a game using these.

I had roles as a designer and teacher in the study. I was the mathematics teacher of the two fifth-grade classes in the 2021-2022 academic year. I have known students since the beginning of the school year. I also designed the Functional Thinking Game that involved functional relationships. I observed the participants when they played the game and conducted the tests and the interviews. I transcribed and analyzed the data through qualitative method procedures.

3.8 Validity and Reliability

Fraenkel et al. (2012) defined validity and reliability as “Validity refers to the appropriateness, meaningfulness, correctness, and usefulness of the inferences a researcher makes. Reliability refers to the consistency of scores or answers from one administration of an instrument to another, and from one set of items to another” (Fraenkel et al., 2012, p. 147). The items in the FTT instrument were modified from the literature and verified by a mathematics education researcher to help to increase the instrument's content validity. In this study, to provide the accuracy or credibility of the findings, member checking and triangulation methods were used. Triangulation is “the process of corroborating evidence from different individuals (e.g., a principal and a student), types of data (e.g., observational field notes and interviews), or methods of data collection (e.g., documents and interviews) in descriptions and themes in qualitative research” (Creswell, 2012, p. 259). The data for this study was collected from several methods: interviews and worksheets for the FTT solutions.

Creswell (2007) “cross-checking is developed by different researchers by comparing the results that are independently derived” (p. 190). In this study, to provide the reliability of coding the data, cross-checking method was used. Then two coders (the researcher and an expert) worked together to analyze the data. 20% of the participants in the pre-test and one participant for each interview were coded by the expert, these codes were compared, and the codes were discussed and agreed upon. Then, necessary changes were made in the analysis.

3.9 Ethical Consideration

Confidentiality and anonymity are important issues to provide ethical reliability. Before conducting the study, necessary permissions were obtained from the Middle East Technical University Ethics Committee and Mardin Provincial Directorate for National Education (see Appendix C and D). Then, the researcher explained the aim of the study and the data collection process to parents, the school management, and teachers. After these, the written consent forms from the students' parents were collected. The names of the participants were changed to provide confidentiality of them and nicknames were used instead.

CHAPTER 4

FINDINGS

In this chapter, for each student, students' responses regarding generalizing and representing functional thinking for each problem and each sub-question will be described one by one. Findings showing the levels of answers given to the questions for each participant and the forms of representation are presented in the form of a separate table for each question. In this section, the answers of each student, the level of these answers, and the forms of representation they use are explained in detail. The data for the pre-test, pre-interview, post-test, and post-interview will be listed in the tables from left to right. Each one includes three columns. The first column refers to the accuracy of the answers given by the students, the second one refers to the levels of sophistication regarding generalization for functional relationships, and finally, the representations used for this answer. A summary of each student will be presented at the end of the chapter.

4.1 Findings for Students' Level of Sophistications for Generalization Processes of Functional Relationships

The students' generalization processes will be documented in this part of the study. For each student, firstly, the findings for the first problem and then the findings for the second problem will be shared. While making explanations about the findings tables, instead of going one by one for each item, the items that were answered at the same level were grouped and explained together for each student. See Table 3.8 and Table 3.9 in Chapter 3 for detailed codes, explanations, and examples.

4.1.1 Findings for Zeynep

Zeynep's Levels of Sophistication for the first and the second problem were recorded on the table, respectively, and then the findings were reported for each of them in two parts correspondingly.

4.1.1.1 Zeynep's Findings for the First Problem

In this part, findings for the first problem will be reported for Zeynep.

Table 4.1 Zeynep's Generalization Levels for the First Problem

Items	Pre-test		Pre-interview		Post-test		Post-interview	
	Corr	Str	Corr	Str	Corr	Str	Corr	Str
1a	1	-	1	-	1	-	1	-
1b	1	RPP	1	CR	1	RPG	1	FCW
1c	1	FCW	1	FCW	1	FCW	1	FCW
1d	0	RPP	0	SI	1	FCS	1	FCS
1e	1	FR	1	FR	1	FR	1	FR
1f	1	-	1	-	1	-	1	-
1g	0	O	0	O	1	CR	1	FCW
1h	0	FEW	0	O	1	FCW	1	FCW
1i	0	SI	0	FP	0	SI	1	FCS

The generalization levels for Zeynep for the first problem are presented in Table 4.1. According to this table, responses to questions 1a, 1c, 1e, and 1f were all correct and at the same level. As seen above, the answer for question 1b was found at RPP level in the pre-test, CR level in the pre-interview, RPG level in the post-test, and FCW level in the last interview. This shows that the level of the student gradually increased after interacting with the game. While question 1d was answered incorrectly and at the RPP level in the pre-test, he gave a wrong answer again in the pre-interview, but it was coded at the SI level. After interacting with the game, this level in the post-test and post-interview was found as FCS. While 1g was coded as wrong and the other category in the pre-test and pre-interview, a correct answer was given at the CR level in the post-test. In the last interview, the student's response for 1g was coded at the FCW level. While question 1h was answered at the FEW levels in the pre-test and Other category in the pre-interview, it increased to the FCW level in the post-test and post-interview. While question 1i was answered incorrectly and at SI level in the pre-test and post-test, it increased to FP level in the first interview and FCS level in the last interview.

4.1.1.2 Zeynep's Findings for the Second Problem

In this part, findings for the second problem will be reported for Participant 1.

Table 4.2 Zeynep's Generalization Levels for the Second Problem

Items	**Pre-test**		**Pre-interview**		**Post-test**		**Post-interview**	
	Corr	**Str**	**Corr**	**Str**	**Corr**	**Str**	**Corr**	**Str**
2a	0	-	1	-	1	-	1	-
2b	0	RPG	1	CR	1	CR	1	CR
2c	0	CR	0	SI	0	FEW	1	FCW
2d	0	O	0	RPG	0	IFCS	1	FCS
2e	0	IFR	1	FR	1	FR	1	FR

In Table 4.2, the generalization levels for Zeynep for the second problem are presented. While she gave a wrong answer for question 2a in the pre-test, she realized that she made a mistake in the first interview and then created a correct table. Thus, as seen in Table 4.2, she gave a wrong response only for the pre-test; otherwise, after then, it was answered correctly. While question 2b was answered wrongly at the RPG level in the pre-test, it was coded at the CR level in the pre-interview, post-test, and post-interview. While 2c was answered incorrectly at the SI level in the pre-test and pre-interview, an incorrect response was given as the FR strategy in the post-test, and it was increased to the FCW level in the last interview. While 2d was answered in the Other category in the pre-test and RPG level in the pre-interview, her level was found as IFCS in the post-test and increased to FCS level in the last interview. While 2e was answered as the IFR strategy in the pre-test, it changed to the FR strategy in the post-test and the last interview.

4.1.2 Findings for Yavuz

Yavuz's generalization levels for the first and the second problems were recorded, on the table, respectively, and then the findings were reported for each of them in two parts correspondingly.

4.1.2.1 Yavuz's Findings for the First Problem

In this part, findings for the first problem were reported for Yavuz.

Table 4.3 Yavuz's Generalization Levels for the First Problem

Items	**Pre-test**		**Pre-interview**		**Post-test**		**Post-interview**	
	Corr	**Str**	**Corr**	**Str**	**Corr**	**Str**	**Corr**	**Str**
1a	1	-	1	-	1	-	1	-
1b	1	RPG	1	RPG	1	CR	1	RPP
1c	0	CR	1	CR	0	RPG	0	SI
1d	0	NR	0	O	1	FPS	1	FPS
1e	1	FR	1	FR	1	FR	1	FR

Table 4.3 (continued)

Items	Pre-test		Pre-interview		Post-test		Post-interview	
1f	1	-	1	-	1	-	1	-
1g	1	CR	1	RPG	0	NR	1	FP
1h	0	RPG	0	O	0	RS	1	FPS
1i	0	NR	0	NR	1	FPS	1	FPS

The generalization levels for Yavuz for the first problem are presented in Table 4.3. According to this table, responses to questions 1a, 1e, and 1f were all correct and at the same level. The answers given for question 1b were correct and at the RPG level in the pre-test and the first interview, while it was correctly answered at the CR level in the post-test and at the RPP level in the last interview. While question 1c was answered at the CR level in the pre-test and the first interview, it was recorded at the RPG level in the post-test and increased to the SI level in the last interview. Question 1d was not responded to in the pre-test; while his answer was in the Other category in the first interview, he gave a correct answer in the post-test and in the last interview, and the level rose to the FPS.

For 1g, in the pre-test, the answer was correct and at the CR level, correct and at the RPG level in the first interview, but he did not answer the question in the post-test. The response was at the FP level in the last interview. While question 1h was answered at RPG level in the pre-test, his answer was in the Other category in the first interview; his level changed to RS in the post-test and increased to FPS level in the last interview. While he did not respond the question 1i in the pre-test and the first interview, it increased to the FPS level in the post-test and in the last interview. While the highest level he could get was CR in the pre-test and pre-interview, in the post-test and post-interview, the highest level was FPS.

4.1.2.2 Yavuz's Findings for the Second Problem

In this part, findings for the second problem will be reported for Yavuz.

Table 4.4 Yavuz's Levels of Sophistication for the Second Problem

Items	**Pre-test**		**Pre-interview**		**Post-test**		**Post-interview**	
	Corr	**Str**	**Corr**	**Str**	**Corr**	**Str**	**Corr**	**Str**
2a	0	-	0	-	1	-	1	-
2b	0	RPG	1	RPG	1	CR	1	CR
2c	0	CR	0	CR	0	CR	1	FPS
2d	0	NR	0	NR	1	FPS	1	FPS
2e	0	IFR	0	IFR	1	FR	1	FR

Table 4.4 presents the generalization level for Yavuz for the second problem. While question 2a was answered incorrectly in the pre-test and first interview, he answered it correctly in the post-test and final interview. While the answers of 2b were at the RPG level in the pre-test and the first interview, it was increased to CR level in the post-test and post-interview. While the response to question 2c was at the CR level in the pre-test, first interview, and post-test, it was found at the FPS level in the last interview. While question 2d was not responded to in the pre-test and the first interview, it was coded at the FPS level in the post-test and last interview. While 2e was coded as the IFR strategy in the pre-test and first interview, he answered as the FR strategy in the post-test and post-interview. While Yavuz's pre-answers to the second question were found CR, which is the highest level he could, this level changed after his interaction with the game, and the highest level he could get was FPS.

While Yavuz did not answer the questions using symbols or variables at first, leaving them unanswered, after interacting with the game, he started to use symbols meaningfully and answered these questions using symbols in the post-test and in the last interview.

4.1.3 Findings for Harun

Harun's generalization levels for the first and the second problem were recorded on the table, respectively, and then the findings were reported for each of them in two parts correspondingly.

4.1.3.1 Harun's Findings for the First Problem

In this part, findings for the first problem were reported for Harun.

Table 4.5 Harun's Generalization Levels for the First Problem

Items	**Pre-test**		**Pre-interview**		**Post-test**		**Post-interview**	
	Corr	**Str**	**Corr**	**Str**	**Corr**	**Str**	**Corr**	**Str**
1a	1	-	1	-	1	-	1	-
1b	1	RPG	1	CR	1	FCW	1	FCW
1c	0	O	0	FEW	1	FCW	1	FCW
1d	0	O	1	SI	0	FPS	1	FPS
1e	1	FR	1	FR	1	FR	1	FR
1f	1	-	1	-	1	-	1	-
1g	1	FCW	1	FP	1	FCW	1	FCW
1h	0	SI	1	FCW	1	FCW	1	FCW
1i	0	O	0	O	0	SIS	1	FPS

The generalization levels for Harun for the first problem are presented in Table 4.5. As seen in the table, the answers to questions 1a, 1e, and 1f were correct and at the same level. While question 1b was answered correctly in all of them, it was responded at the RPG level in the pre-test and the CR level in the first interview, while it was found at the FCW level in the post-test and last interview. While the response to question 1c was in the Other category in the pre-test, it was answered incorrectly at the FEW level in the first interview and rose to the FCW level in the post-test and last interview. While the response to question 1d was in the Other category in the pre-test, it was answered at the SI level in the first interview and then increased to the FPS level in the post-test and last interview. While question 1g was answered correctly and at the FCW level in the pre-test, post-interview, and post-test, it was responded at the SI level in the first interview. While question 1h was answered at SI level in the pre-test, it was increased to FCW level in the first interview, post-test, and last interview. While answering question 1i in the Other category in the pre-test and first interview, it was responded incorrectly at the SIS level in the post-test, and an increment was shown in the last interview by answering it correctly at the FPS level. In the pre-test and pre-interview, his answers varied from the RPG level to the FCW level, which was the highest level he could get. After interacting with the game, in the post-test and post-interview, the highest level he could reach was FCW again, but the answers at this level showed intensity.

4.1.3.2 Harun's Findings for the Second Problem

In this part, findings for the second problem will be reported for Harun.

Table 4.6 Harun's Generalization Levels for the Second Problem

Items	**Pre-test**		**Pre-interview**		**Post-test**		**Post-interview**	
	Corr	**Str**	**Corr**	**Str**	**Corr**	**Str**	**Corr**	**Str**
2a	1	-	1	-	1	-	1	-
2b	1	CR	1	RPG	1	RPG	1	CR
2c	0	CR	0	CR	0	RPG	1	RPG
2d	0	O	1	CR	0	SIS	1	FPS
2e	0	IFR	1	IFR	1	FR	1	FR

Table 4.6 presents the generalization levels for Harun for the second problem. As seen in the table, he answered question 2a correctly in all of them. While question 2b was answered at CR level in the pre-test, it was responded correctly at RPG level in the first interview and post-test and changed to CR level in the last interview. While 2c was answered correctly at the CR level in the pre-test and the first interview, it was also answered correctly but at the RPG level in the post-test and post-interview. While question 2d was answered in the Other category in the pre-test, it was responded correctly at the CR level in the first interview, while it was answered incorrectly and at the SIS level in the post-test, it was coded at the FPS level correctly in the post-interview. While question 2e was the IFR strategy in the pre-test and first interview, he answered it correctly as the FR strategy in the post-test and final interview. In the second question, the highest level that Harun frequently used and could reach was the CR level, while the highest level he could get after interacting with the game changed to FPS.

4.1.4 Findings for Funda

Funda's generalization levels for the first and the second problem were recorded on the table, respectively, and then the findings were reported for each of them in two parts correspondingly.

4.1.4.1 Funda's Findings for the First Problem

In this part, findings for the first problem will be reported for Funda.

Table 4.7 Funda's Generalization Levels for the First Problem

Items	**Pre-test**		**Pre-interview**		**Post-test**		**Post-interview**	
	Corr	**Str**	**Corr**	**Str**	**Corr**	**Str**	**Corr**	**Str**
1a	1	-	1	-	1	-	1	-
1b	1	RPG	1	RPG	1	RPG	1	RPG
1c	0	RS	0	RPG	0	RPG	0	CR
1d	0	NR	0	RPG	0	RPG	0	FP
1e	1	FR	1	FR	1	FR	1	FR
1f	1	-	1	-	1	-	1	-
1g	1	RPG	1	FP	1	RPG	1	RPG
1h	0	O	0	O	0	RPG	0	FEW
1i	0	NR	0	RPG	0	RPG	0	FP

Table 4.7 presents generalization levels for Funda for the first problem. As seen in the table, questions 1a, 1b, 1e, and 1f were answered correctly and at the same level. While the answer to question 1c was at the RS level in the pre-test, it was answered at the RPG level in the first interview and post-test and increased to the CR level in

the last interview. All 1d questions were answered incorrectly. While he did not answer in the pre-test, he answered at the RPG level in the first interview and post-test and rose to the FP level in the last interview. While question 1g was responded correctly at the RPG level in the pre-test, post-test, and last interview, it was responded at the FP level in the first interview. While the response of 1h was in the Other category in the pre-test and first interview, it was answered incorrectly at the RPG level in the post-test and increased to the FEW level, but was still incorrect in the last interview. While question 1i was not answered in the pre-test, it was answered incorrectly, and at the RPG level in the first interview and post-test, it increased to the FEW level but was still incorrect in the last interview.

4.1.4.2 Funda's Findings for the Second Problem

In this part, findings for the second problem will be reported for Participant 4.

Table 4.8 Funda's Generalization Levels for the Second Problem

Items	**Pre-test**		**Pre-interview**		**Post-test**		**Post-interview**	
	Corr	**Str**	**Corr**	**Str**	**Corr**	**Str**	**Corr**	**Str**
2a	1	-	1	-	1	-	1	-
2b	1	RPG	1	RPG	1	RPG	1	RPG
2c	0	O	1	RPG	0	O	0	FP
2d	0	NR	0	RPG	0	NR	0	FP
2e	0	IFR	0	IFR	0	IFR	1	FR

Generalization levels for Funda for the second problem are presented in Table 4.8. As seen in this table, questions 2a and 2b were answered correctly and at the same level. While the answer of 2c was in the Other category in the pre-test, it was answered wrongly and at the RPG level in the first interview and at the Other category in the post-test and increased to the FP level but was still incorrect in the last interview. Question 2d was not answered in the pre-test and post-test; while it was answered at the RPG level and was incorrect in the first interview, it boosted to the FP level, still not correct in the last interview. While the response of 2e was IFR in the pre-test, first interview, and post-test, it changed to correct and FR in the last interview. While Funda's answers showed intensity at the RPG level as the highest level she could reach the beginning, the FP level was frequently encountered as the highest level she could reach after interacting with the game.

4.2 Findings for Student’ Representation Processes of Functional Relationships

In this part, the findings related to the second research question will be documented. This section referred to students' representation processes for functional relationships. In questions 1a, 2a and 1f, participants were expected to fill in the given table appropriately. Since this task did not require using any representation forms, these parts were left blank in the tables below. In addition, the participants were asked to explain the relationships between the variables verbally (in items 1c, 1h, and 2c) and ask to explain the patterns in the tables (in items 1b, 1g, and 2b), the verbal representation type was generally used in these options. Since the participants in 1d, 2d and 1i were asked to write the rule that explains the relationships between the variables by using symbols or variables, the representations of the answers in this part varied. When the responses of the students were examined, all participants were found to use the verbal expression representation type extensively while answering the questions 1b, 1c, 1g, 1h, 2b, and 2c. In addition to this representation, some also used numerical representation in some of these questions. In questions 1e and 2e, the students were asked to find the number of people and the number of leaves in the 100^{th} step, respectively. Thus, mostly numeric and verbal representation forms were coded in the students’ responses. In questions 1a, 1f, and 2a, all participants filled in the given table correctly or created the desired table correctly. Only Yavuz created a table in question 2a in the pre-test but filled it out incorrectly. This shows that almost all participants had no problem with tabular representation type. For each participant, the representation processes will be reported separately in the following parts.

4.2.1 Findings for Zeynep

Zeynep's representation processes for the first and the second problem were recorded on the table, respectively, and then the findings were reported for each of them correspondingly.

Table 4.9 Zeynep's Representations for the First Problem

Items	Pre-test	Pre-interview	Post-test	Post-interview
1a	-	-	-	-
1b	V	V	V	V
1c	V	V	V	V
1d	PN	PN	S	VS
1e	VN	VN	VN	V
1f	-	-	-	-
1g	V	V	V	VN
1h	V	V	V	VN
1i	NP	VNP	NP	VS

In items 1b, 1c, 1g, and 1h, Zeynep used verbal representation; additionally, she used numerical form in items 1g and 1h in the post-interview. While responding to questions 1d and 1i, Zeynep used numerical or pictorial representation in addition to verbal representation in the pre-test and first interview. In contrast, he used symbolic representation in addition to verbal and numerical representation in the post-test and final interview. It is seen in Table 4.9 that he used verbal and numerical representation forms in question 1e.

Table 4.10 Zeynep's Representations for the Second Problem

Items	Pre-test	Pre-interview	Post-test	Post-interview
2a	-	-	-	-
2b	V	VNT	V	V
2c	V	V	V	VN
2d	P	PV	SN	VSN
2e	VN	VN	N	VN

As seen in Table 4.10, in the second problem, Zeynep answered questions 2b and 2c by using verbal representation; additionally, she used numeric and tabular for 2b in the pre-interview and numeric for 2c in the post-interview, while she used mostly verbal and numeric representation forms in 2e. In item 2d, Zeynep's responses on the pre-test and pre-interview were coded as pictorial while it was found as symbolic and numeric on the post-test and post-interview (additionally verbal representation form was used in the pre-interview and post-interview). Her response for item 2d of the pre-test was shown in Figure 4.1. As seen below, she explained the relationship between the number of days and the number of leaves by drawing circles and rectangles.

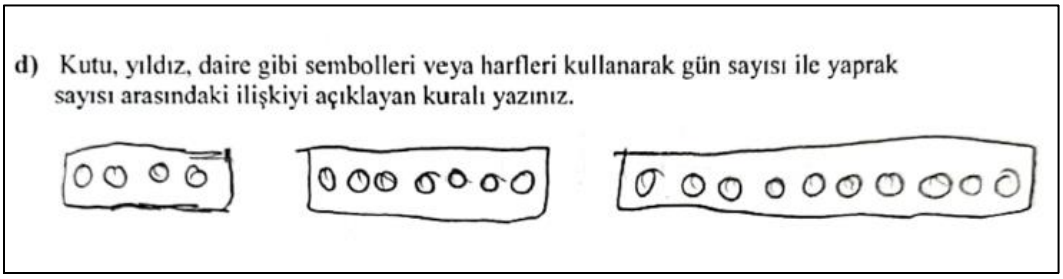


Figure 4.1. Zeynep's Pictorial Representation Form for Item 2d in the Pre-Test

In Figure 4.2, Zeynep's representation form for Item 2d in the post-test was presented. She expressed the number of days using a box and the number of leaves using a circle symbol. Using these symbols, she wrote the equation showing the relationship between the number of days and the number of leaves incorrectly.

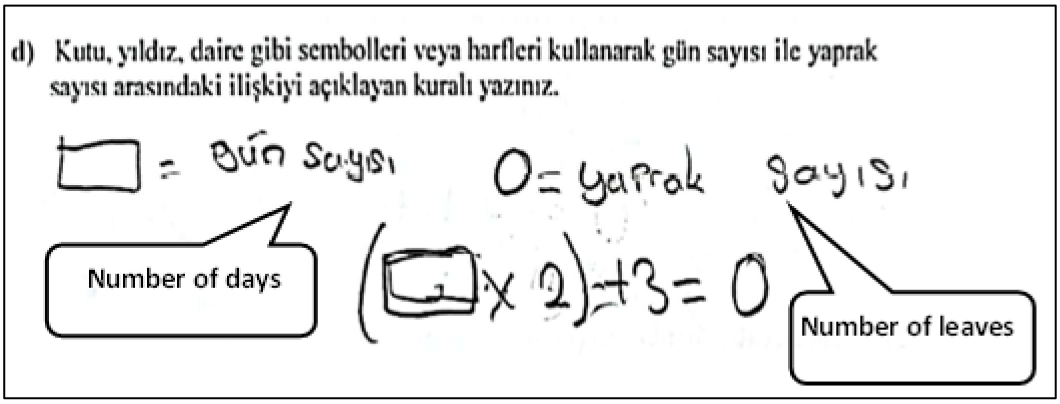


Figure 4.2. Zeynep’s Symbolic and Numeric Representation Form for Item 2d in the Post-Test

When she was asked about her answer at the last interview, the student reconsidered the relationship between the variables and, noticing her mistake, wrote down the correct answer (see Figure 4.3).

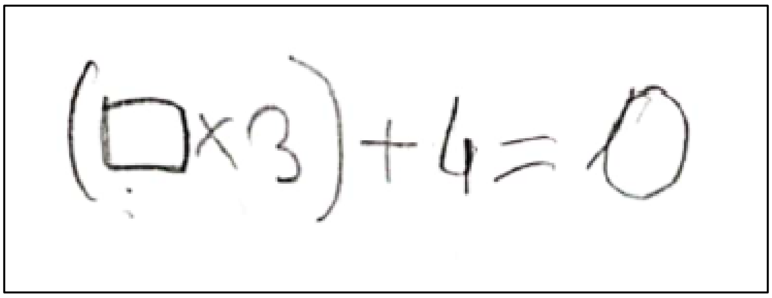


Figure 4.3. Zeynep’s Symbolic and Numeric Representation form for Item 2d in the Post-Interview

In addition to these, regarding the use of representations, while Zeynep did not use variables at the beginning and only used symbols for visualization purposes, after interacting with the game, she was found to use symbols correctly and meaningfully and in different ways by using commutative and associative properties. In Figure 4.4, Zeynep’s representation form for the function rule about the game was shown. In Zeynep's answer, the heart symbol represents the starting point, the box represents the number of hits, the star represents the score obtained from one accurate shot, and the circle symbol represents the total score.

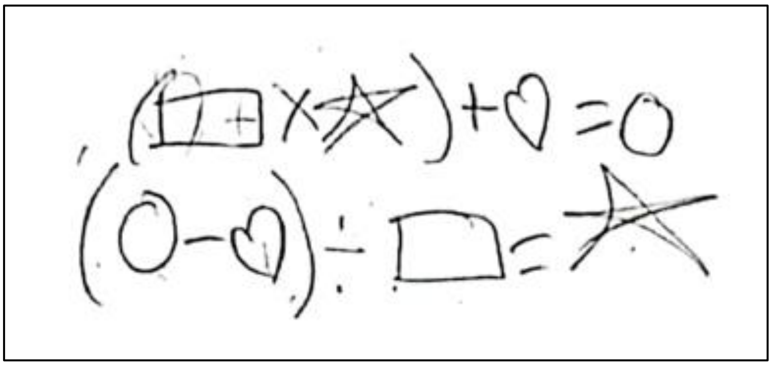


Figure 4.4. Zeynep’s Symbolic Representation form for the rule of the Game

4.2.2 Findings for Yavuz

In Table 4.11, Yavuz's representations for the first problem are presented. In items 1b, 1c, 1g, and 1h, Yavuz used intensively verbal representation; additionally, he used numerical form in items 1c and 1g, and symbolic form in 1h in the post-interview. While answering questions 1d and 1i Yavuz did not use any forms of representation in the pre-test, he used verbal and tabular representation forms in item 1d only in the pre-interview; he was found to use symbolic and numerical representations in the post-test and symbolic, numerical, and verbal representations in the post-interview. It was seen that he used verbal and numerical representation forms in question 1e.

Table 4.11 Yavuz's Representations for the First Problem

Items	Pre-test	Pre-interview	Post-test	Post-interview
1a	-	-	-	-
1b	V	V	V	V
1c	V	V	V	VN
1d	NRep	VT	SN	VSN
1e	VN	V	VN	VN
1f	-	-	-	-
1g	V	V	NRep	VN
1h	V	V	V	VS
1i	NRep	NRep	SN	VSN

In the post-test, he used symbolic and numeric representations for item 1i. His response is shown in Figure 4.5. He wrote the rule which explains the relationship between the number of tables and the number of people by using the box symbol in a particular way; that is, he indicated the numbers above the boxes.

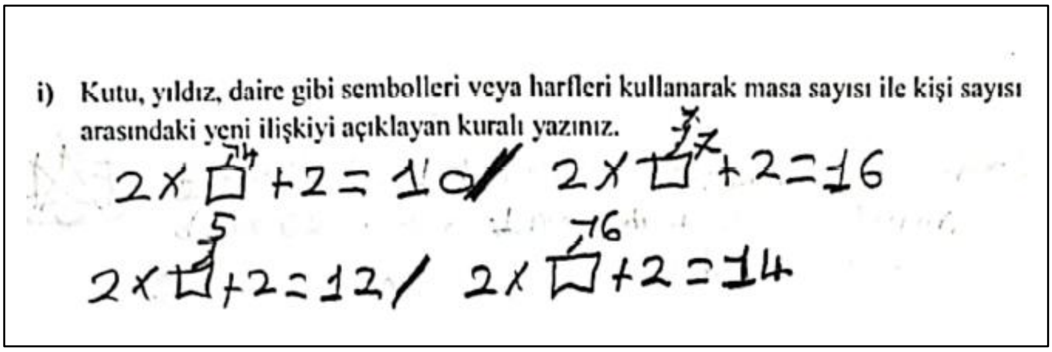


Figure 4.5. Yavuz's Symbolic and Numeric Representation forms for Item 1i in the Post-Test

Similarly, in the second problem, he used verbal representation for items 2b and 2c; additionally, he used symbolic and numeric representations for item 2c in the post-interview (see Table 4.12). Also, for item 2d, he did not answer in the pre-test and pre-interview, while he responded by using symbolic and numeric representation forms in the post-test and post-interview; in addition, he used verbal form in the post-interview. Finally, he used verbal and numeric forms to answer item 2e.

Table 4.12 Yavuz's Representations for the Second Problem

Items	Pre-test	Pre-interview	Post-test	Post-interview
2a	-	-	-	-
2b	V	V	V	V
2c	V	V	V	VSN
2d	NRep	NRep	SN	VSN
2e	VN	VN	VN	VN

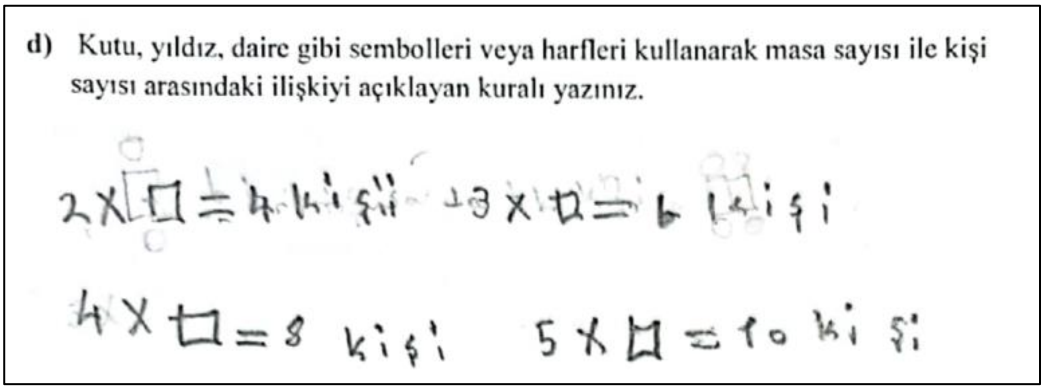
4.2.3 Findings for Harun

Table 4.13 Harun's Representations for the First Problem

Items	Pre-test	Pre-interview	Post-test	Post-interview
1a	-	-	-	-
1b	V	V	V	V
1c	V	V	V	V
1d	SN	SN	VSNP	VS
1e	VN	VN	VN	VN
1f	-	-	-	-
1g	V	VN	V	V
1h	V	V	V	V
1i	SN	VSN	VSNP	VSN

In Table 4.13, Harun's representations for the first problem are presented. His answers for the items 1b, 1c, 1g, and 1h were coded as verbal. In addition to this, he used a numeric form in item 1g in the pre-interview. While Harun answered question 1d, he used symbolic and numerical representations in the pre-test, pre-interview, and post-test (additionally pictorial form was used in the post-test) while verbal and symbolic representations were included in the post-interview.

In item 1i, his responses were coded as verbal, symbolic, and numeric forms in all implementations except for the pre-test; additionally, a pictorial form was also included in the post-test. In addition to these, he used verbal and numerical representation forms together in item 1e.


d) Kutu, yıldız, daire gibi sembolleri veya harfleri kullanarak masa sayısı ile kişi sayısı arasındaki ilişkiyi açıklayan kuralı yazınız.

2 X □ = 4 kişi 3 X □ = 6 kişi

4 X □ = 8 kişi 5 X □ = 10 kişi

Figure 4.6. Harun’s Symbolic and Numeric Representation forms for Item 1d in the Pre-Test

Harun’s symbolic and numeric representation forms for item 1d in the pre-test were shown in Figure 4.6. Harun expressed the number of tables using numbers while expressing the coefficient in the problem with a box symbol in item 1d in the pre-test.

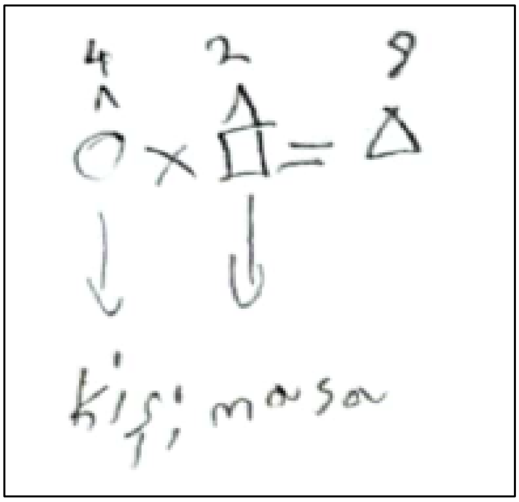


Figure 4.7. Harun’s Symbolic and Numeric Representation forms for Item 1d in the Pre-Interview

In the pre-interview, while writing the equation, he used different symbols for each value and stated the values that each of them expressed by writing. He used the circle symbol for the number of people sitting at a table, the box for the number of tables, and the triangle for the total number of people (see Figure 4.7).

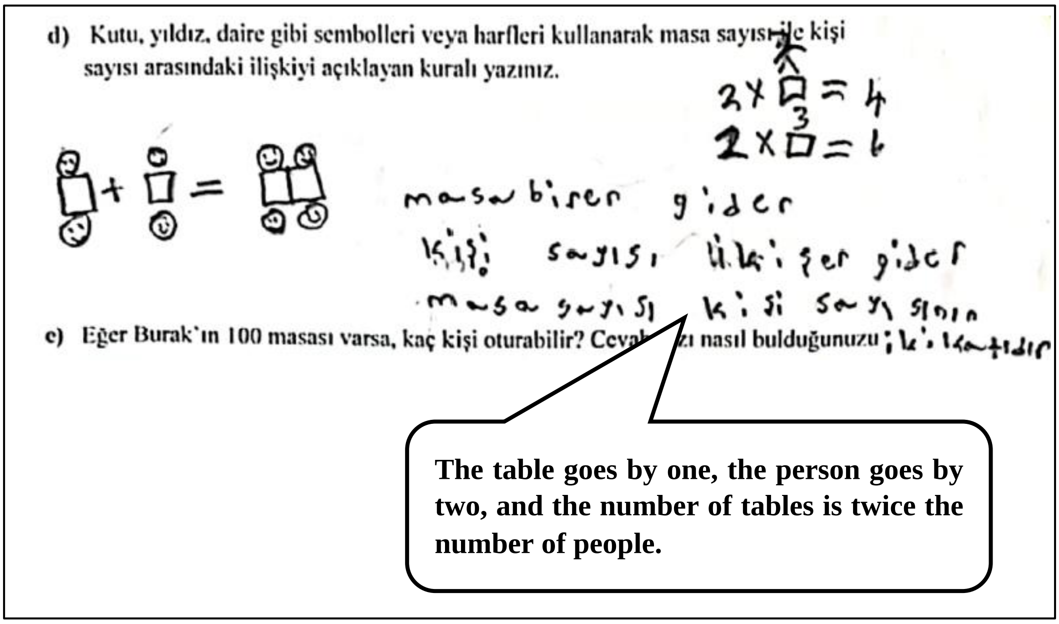


Figure 4.8. Harun’s VSNP Representation forms for Item 1d in the Post-Test

Figure 4.8 shows his answer in the post-test. While expressing the number of people sitting at each table and the total number of people numerically, he expressed the number of tables with a box symbol, gave an example for two different table numbers, and wrote the value of the symbol on the boxes.

Table 4.14 Harun's Representations for the Second Problem

Items	**Pre-test**	**Pre-interview**	**Post-test**	**Post-interview**
2a	-	-	-	-
2b	V	V	V	V
2c	V	V	V	VSN
2d	SN	VT	SN	VSN
2e	VN	VN	VN	VN

Similar to the first question, he answered 2b and 2c verbally; additionally, he used symbolic and numeric representations in item 1c in the post-interview. In 2d, while he used symbolic and numeric representations in the pre-test, post-test, and post-interview, he additionally used verbal form in the post-interview. Also, in the pre-interview, verbal and tabular forms were used for item 2d. Finally, In item 2e, where students were asked to find the total number of leaves on the 100th day, Harun used verbal and numeric representations in all implementations.

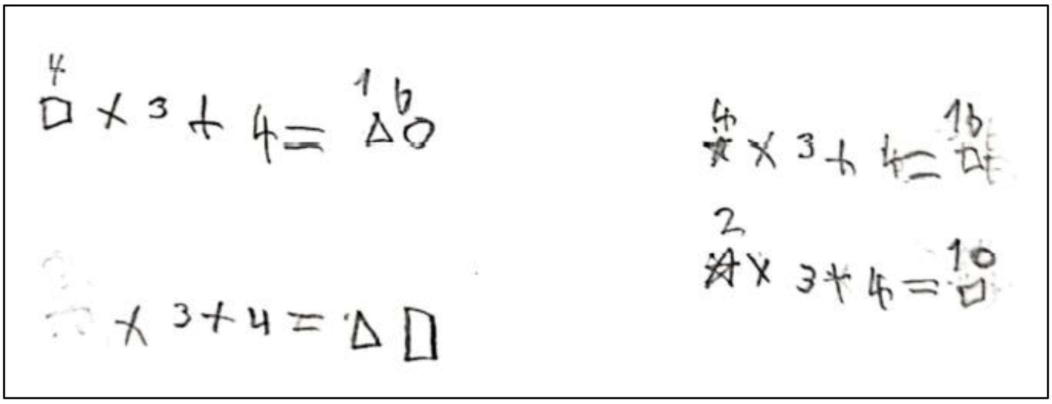


Figure 4.9. Harun’s Symbolic and Numeric Representation forms for Item 2d in the post-Interview

Figure 4.9 shows the answer given by the Harun to question 2d in the last interview. Here, firstly, he expressed the number of days and the total number of sprouting leaves using different symbols such as box, triangle, and circle for each number, while writing the number of leaves sprouting in one day numerically as a coefficient of 3 as seen on the left of Figure 4.8. In the continuation of the interview, instead of using different symbols for each number, he indicated the number of days with a star and the total number of leaves with a box as seen on the right of Figure 4.8.

4.2.4 Findings for Funda

Funda’s representations for the first problem are presented in Table 4.15. While Funda did not use any representation in the pre-test while answering items 1d, and 1i (giving no response), she used both symbolic and verbal representations in the pre-interview; the symbolic form was used in the post-test; also tabular form was used in the 1d item. For the post-interview, she used verbal, symbolic, and numeric forms for item 1i, while she only used verbal forms for item 1d. In addition, she used verbal and numerical representations together in item 1e.

Table 4.15 Funda’s Representations for the First Problem

Items	Pre-test	Pre-interview	Post-test	Post-interview
1a	-	-	-	-
1b	VN	V	VN	V
1c	V	V	V	V
1d	NRep	SV	ST	V
1e	VN	VN	VN	VN
1f	-	-	-	-
1g	VN	V	VN	V
1h	V	V	V	VN
1i	NRep	SV	S	VSN

In the post-test, she answered using a tabular form for item 1d, inserting symbols instead of numbers. An example of this is shown in Figure 4.10.

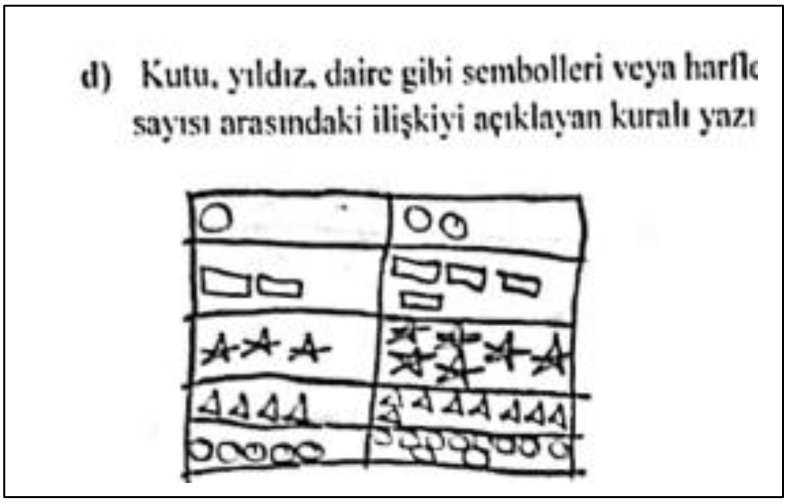


Figure 4.10. Funda’s Tabular and Symbolic Representation Forms for Item 1d in the Post-Test

Funda’s Representations for the Second Problem was presented in Table 4.6. Similarly, in the second problem, in item 2d, the question remained unanswered in the pre-test and post-test, and then the relationship between the number of days and the number of leaves was shown in the form of a table by using symbols and also in numeric and verbal forms. In other items of the problem, mostly verbal and numerical representation forms were used together.

Table 4.16 Funda’s Representations for the Second Problem

Items	Pre-test	Pre-interview	Post-test	Post-interview
2a	-	-	-	-
2b	VN	VN	VN	VN
2c	V	VN	V	VN
2d	NRep	TVSN	NRep	TVSN
2e	VN	VN	N	VN

In the last interview, when she was asked to show the relationship between the number of days and the number of leaves in different ways, he calculated it numerically for different days and stated that this was the most practical way instead of showing it with symbols as seen in Figure 4.11.

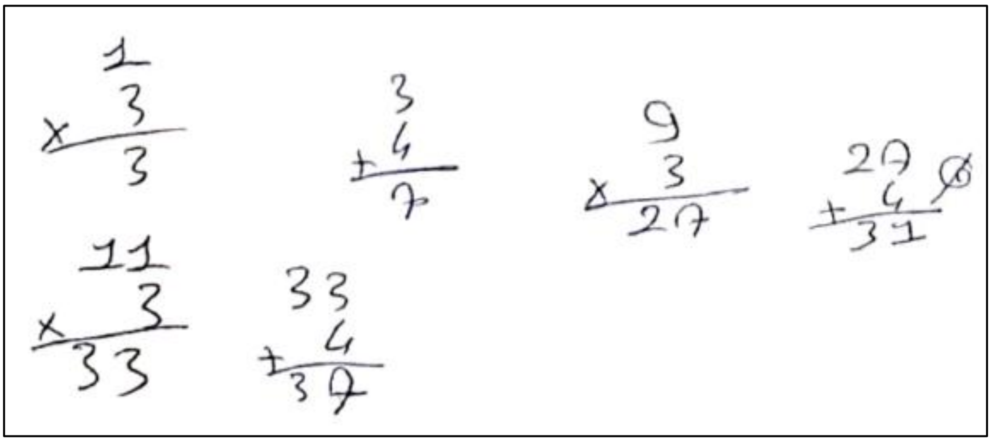


Figure 4.11. Funda’s Numeric Representation form for Item 2d in the Post-Interview

In the light of all these, it can be concluded that all the participants generally used verbal and numerical representations together while answering items 1e and 2e, where students were asked to find the number of people and the total number of leaves on the 100th day respectively. In addition, all participants mostly used verbal, symbolic, and numerical representations together in the last interview in items 1d, 1i, and 2d. In items 1c, 1h, and 2c, where the participants were asked to explain the relationships between the variables verbally, and in items 1b, 1g, and 2b, where they were expected to explain the patterns in the tables, verbal and numeric representation types were generally used.

4.3 Findings Regarding Observations Related to the Functional Thinking Game

Before starting the game, the researcher introduced the game to each student, and the students were asked to carefully read the instructions in the introduction part of the game. The scoreboard in the game was emphasized, and they were asked to carefully look at these tables during the game and observe the changes for each shot. They were asked to think aloud while shooting and share their comments about the progress of the game with the researcher. At the end of each episode, students were asked to make an overall assessment of the game's process. After the game, the participants were asked six questions parallel to the game; three addressed the y=mx functional relationship parallel to the first part of the game, while the remaining three were for the y=mx+n functional relationship parallel to the second part of the game. For each of these six questions, four follow-up questions, which were given in Table 3.5 in Chapter 3, were asked to students. While creating the coding table for student answers, the most sophisticated strategy used throughout the problem was coded instead of coding the strategy for each follow-up question separately. Also, all representations of the students' procedures were coded.

In addition to these questions, one question, which was named 2d in the second part, was asked to students. In this question, students were expected to generalize the game rule for the given further steps. When students were asked the question, “What is the game rule to get 100 points in total with 100 hits and 100 misses?” (van den Heuvel-Panhuizen et al., 2013, p. 290), their strategies and representation forms were also provided in the tables.

4.3.1 Findings for Zeynep

Zeynep had a little trouble hitting the target board at first while playing the game but soon adapted to this and started shooting more carefully. Here are some of his thoughts on her shots at the levels:

T: Well, can you explain to me the relationships in the scoreboard?

Z: Missed shots are 20 points, and my hits are 20 points right now. Because I shot five, each shot is 4 points. [In here, she means that she had five missed shots and five hits, a total of 20 shots. Since every hit gets 4 points, she had 20 points]. This means that 4 times 5 equals 20, totaling 20 points. In missed shots, the total score does not change, but I lose points; it increases my total shots.

T: What do you mean by losing points?

Z: It's not losing points; it's actually losing shots; by that, I mean I could 5 out of 5, but I shot more. This also affected the total. Each accurate shot gives 3 points. This means that 10 shots will be required because my target score is 30.

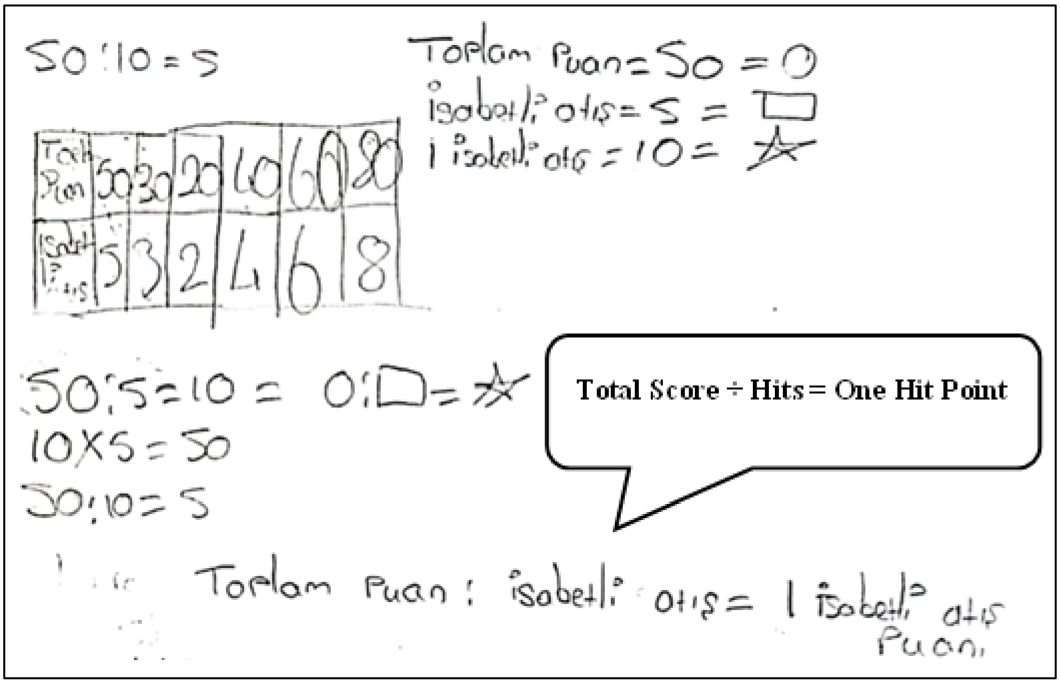


Figure 4.12. Zeynep's Symbolic, Tabular and Numerical Representation forms for Questions related to the Game-Interview

After the game, Zeynep was asked questions parallel to the game. These problems were given under the title of observation protocols in Table 3.5 in Chapter Three. Zeynep's worksheet for the first question is presented in Figure 4.12. She showed the number of hits and the total score in the table and explained the relationship between them as follows:

T: Can you describe to me the relationship between the hit and the total score in letters or symbols?

Z: Teacher, if we divide the hit and the total score, we get the number of points for a hit. If we divide it by the total score, we get the number of hits. If we multiply the number of hits with one hit point, we get the total score.

T: Well, can you tell me what the circle means here?

Z: The circle represents the total score

T: What does the box represent?

Z: Number of hits

T: What does the star mean?

Z: The score in one fixed shot, i.e., 10 points

T: Can you explain to me what the varying quantities are here?

P: The total score and the number of hits because if we do it according to the table, the score of a hit has not changed; only these two have changed.

T: So, what are the constant quantities here?

P: The fixed one is the score of one shot, namely 10. Maybe it would be incomprehensible if I didn’t introduce all of them separately, so I wrote down the names of all of them.

Table 4.17 Zeynep’s Findings for the Game-Interview

Items	**Correctness**	**Strategy**	**Representation**
1a	1	FCW	VSNT
1b	1	FCW	VSNT
1c	1	FCW	VSN
2a	1	FCW	VSNT
2b	1	FCW	VSNT
2c	1	FCW	VSN
2d	1	FR	VN

As seen in Table 4.17, Zeynep answered all questions in the game interview correctly using verbal, symbolic numeric, and tabular representations. It was coded as Functional Condensed with Words which was the highest level to be reached.

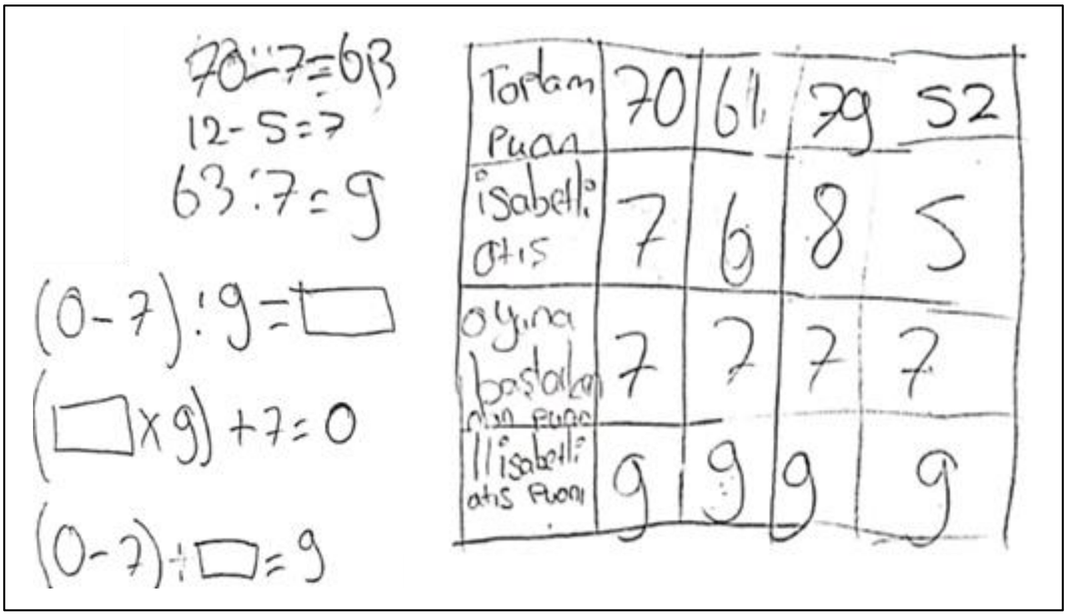


Figure 4.13. Zeynep's Tabular, Symbolic and Numeric Representation Forms for Question 6 in the Game-Interview

Figure 4.13 presents Zeynep's solution way for the sixth question. Zeynep answered the sixth question in this way:

T: Can you explain your procedures to me?

Z: Teacher, I subtracted the starting score from the total score, and I found 63. The reason why I subtracted 5 from 12 is that he made 12 shots, so this is the total shot he made, and he missed 5 of them, that is, he made 7 accurate shots. So, by dividing 63 by 7, I got 9.

T: Well, can you show the relationship between the number of hits and the total score with symbols or letters?

Z: It's the same thing as the previous ones, but I said it's a little different… So, what I mean is that 7 and 9 are always there, I didn't do anything to them, so I wrote the others with symbols like this.

T: What is the reason for writing these in this way?

Z: Teacher, because 7 and 9 are different… I mean, how can I say, others change, but they are always the same in all of them.

T: So, what are the changing quantities here?

Z: The changes are the total score and the number of hits. Fixed values are the score at the start of the game and the score for one hit.

4.3.2 Findings for Yavuz

Yavuz completed the levels of the game, although he had some difficulty shooting at first. During the game and the transitions between levels, he correctly explained the values on the scoreboard, the minimum number of shots needed to reach the target score, and the relationships between the number of shots and the scores.

After the game, Yavuz was asked questions parallel to the game, and he answered all of them correctly. In Table 4.18, Yavuz's findings for the game-interview were shown. He answered all of the questions by using verbal, numeric, and tabular representations. In addition to these, he also used symbolic representation while answering the questions parallel to the second part of the game. Besides, the strategies used by the Yavuz varied from the level of RPG to the level of FPS.

Table 4.18 Yavuz's Findings for the Game-Interview

Items	Correctness	Strategy	Representation
1a	1	FP	TVN
1b	1	SI/FP	TVN
1c	1	RPG	TVN
2a	1	SIS	TVNS

Tablo 4.18 (continued)

Items	Correctness	Strategy	Representation
2b	1	SIS	TVNS
2c	1	FPS	TVNS
2d	1	FR	TVN

The response to the first question of the Yavuz is presented in Figure 4.14. He showed the number of hits and the total score in the table and explained the relationship between them as follows:

T: Can you explain what you think?

Y: Each accurate shot gets 7 points, for example, 7 points in 1 shot, 14 in 2 shots, like 21 in 3 shots, it goes up to 56, so at the end of this, we get 56 in 8 shots.

T: How did you know it was 8?

Y: Because it increases 7 by 7, we need to shoot 8 shots until 56, 8 x 7 makes 56

T: What is the relationship between hit and score?

Y: (writes)

T: Well, what do you think is our rule?

Y: You will multiply the number of hits by 7; for example, someone like 1×7=7 2 x 7=14...

T: Well, can you show me this with symbols?

Y: (thinking…)…(silence)

T: What are the varying quantities?

Y: The score and the number of hits are changing, It is also fixed that we always multiply by 7...

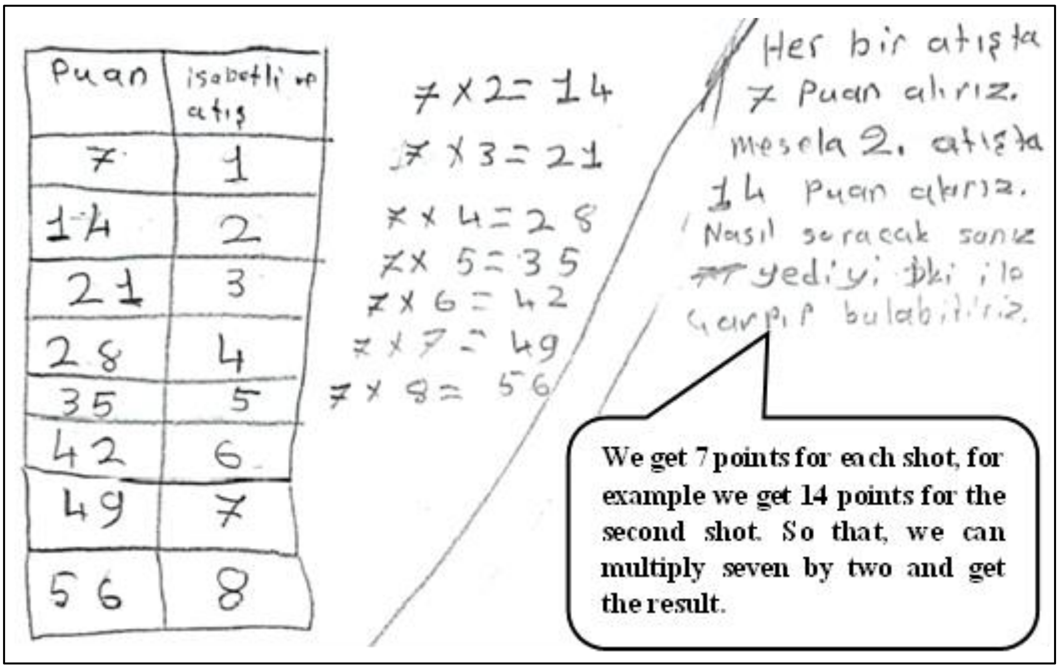


Figure 4.14. Yavuz’s Tabular, Numeric and Verbal Representation Forms for Question 1 in the Game-Interview

Figure 4.15 presents Yavuz’s solution way for the fifth question. Yavuz answered the fifth question in this way:

T: Can you explain your answer to me? How did you find it?

Y: It gives our 4 points at the beginning. When we count five by five, there are 8 until 40; it becomes 40 + 4 = 44

T: What is the relationship between total points and the number of hits?

Y: Since our score increased five by five, I did as follows. Since 5 x 4 = 20, I wrote 4 as a square. Since it was +4 at the beginning, our answer is 20 plus 4 equals 24.

T: So why did you write 4 as a square?

Y: The square could be any number other than 4.

T: What else could it be?

Y: It will be the same operation, it may be 3 or 5 instead of 4, but we will do the same operation again.

T: What does the square represent here?

Y: Number of hits

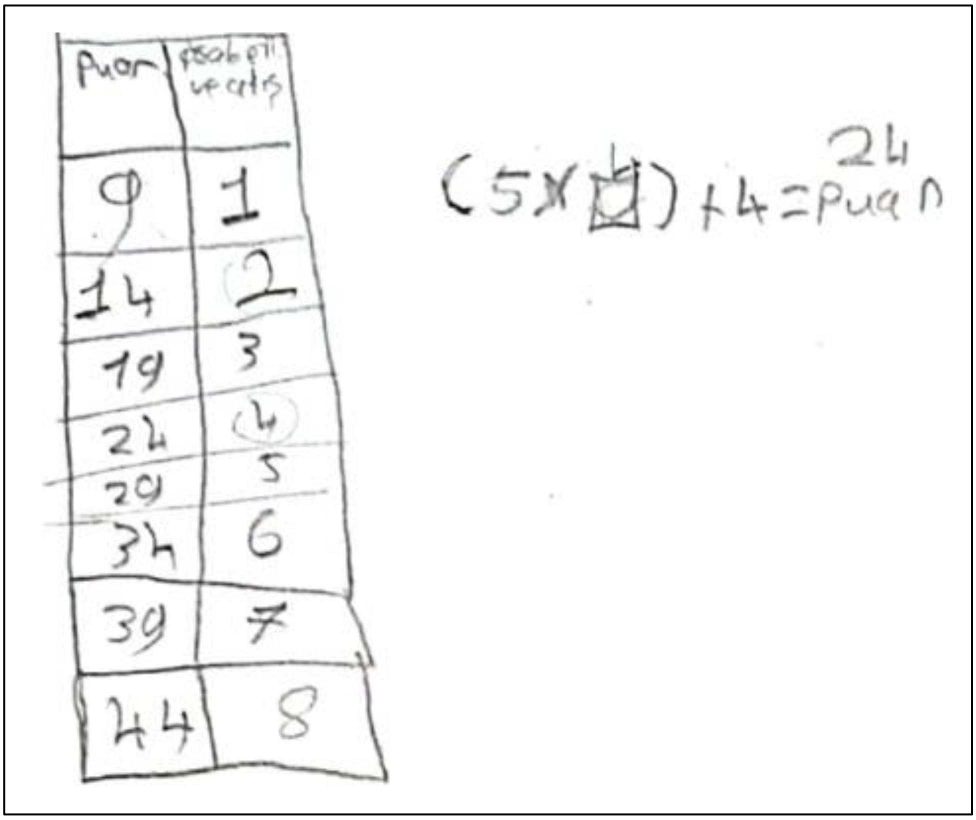


Figure 4.15. Yavuz Tabular, Symbolic, and Numeric Representation Forms for Question 5 in the Game-Interview

4.3.3 Findings for Harun

Harun has completed the levels of the game, although he had some difficulty shooting at first. During the game and the transitions between levels, he correctly explained the values on the scoreboard, the minimum number of hits needed to reach the target score, and the relationships between the number of hits and the scores. Harun correctly mentioned the relationships between the variables while talking about the levels and rules of the game after the game. For example, he stated that the total number of shots increased as a result of the increase in missed and accurate shots. At the same time, while talking about the first part of the game, he showed the relationship between the number of hits and the total score numerically and, unlike the other students, by drawing graphs instead of tables. The column chart drawn by Harun for the relationship between the variables in the first part of the game is given in Figure 4.16.

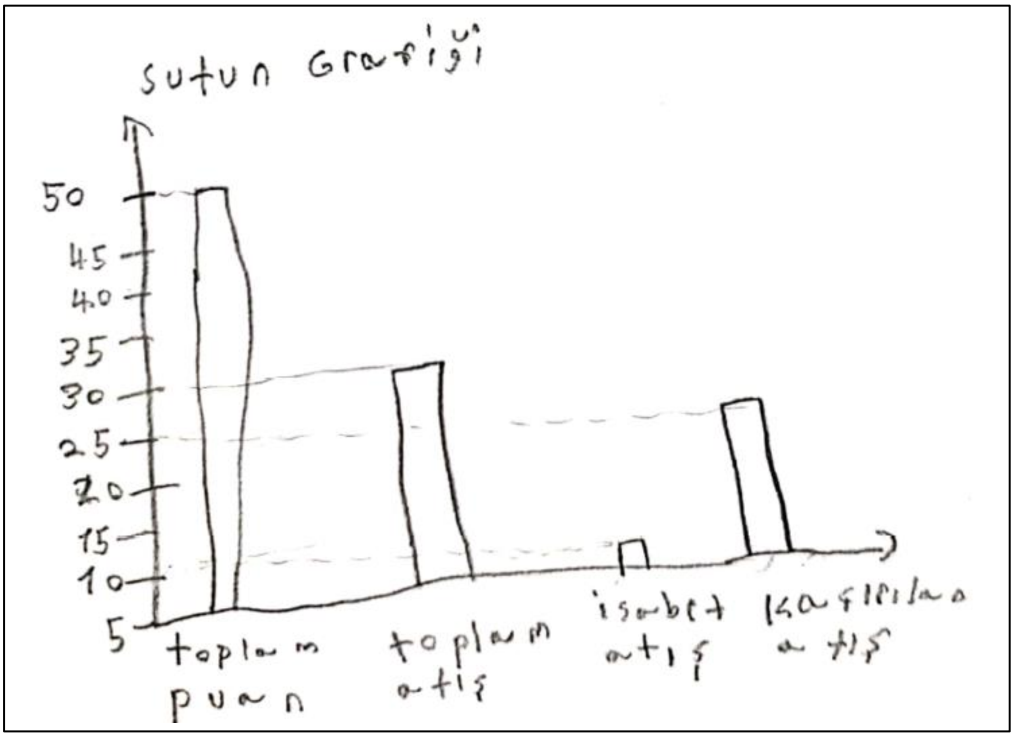


Figure 4.16. Harun's Graphical Representation Forms for the Game-İnterview

After the game, Harun was asked questions parallel to the game, and he answered all of them correctly. In Table 4.19, Harun's findings for the interview on the Functional Thinking Game were shown. He answered all the questions by using verbal, numeric, symbolic, and tabular representations. Besides, the strategies used by the Harun varied from the level of RPG to the level of FEW. During the coding, as stated, the most sophisticated level of Harun's response to each question was coded. For Harun, some responses also included symbols such as at the SIS and FPS levels.

Table 4.19 Harun's Findings for the Game-Interview

Items	Correctness	Strategy	Representation
1a	1	FP	TVNS
1b	1	RPG	TVNS
1c	1	FP	TVNS
2a	1	FEW	TVNS
2b	1	FEW	TVNS
2c	1	FEW	TVNS
2d	1	FR	VN

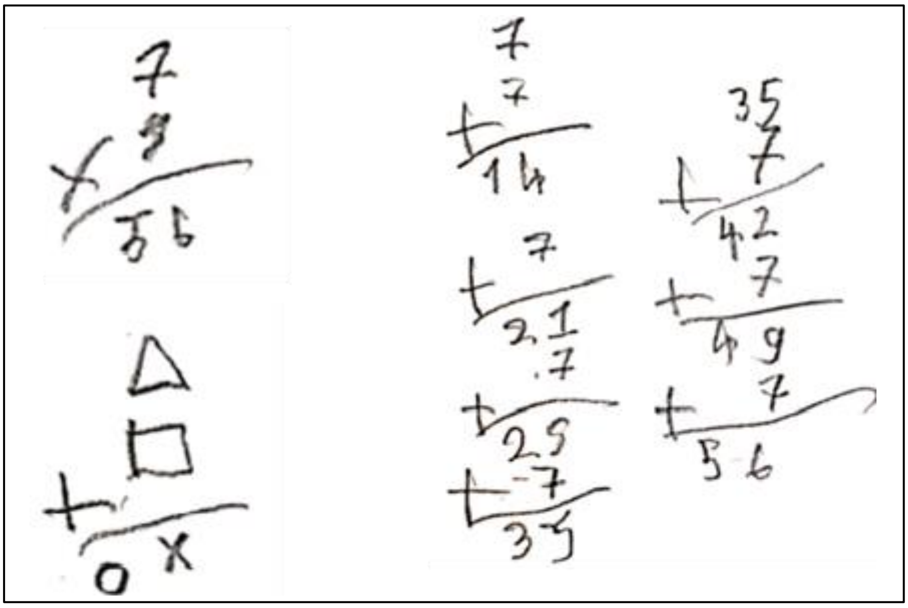


Figure 4.17. Harun’s Numerical Representation Form for Question 1 in the Game-Interview

Figure 4.17 presents Harun's solution way for the first question. While Harun was solving the question with symbols, he stated that he would determine a separate symbol for each number from 0 to 9 and use them for varying quantities while operating. Harun's answers to the first question were as follows:

H: Teacher, we're going to do it like this... We're going to divide 56 by 7 because I thought whatever we multiply by 7 would be 56. It takes 8 for us to reach 56. Teacher, if I do less than 8, it will be 49, and it will be 42. However, for example, if I made 8 hits and missed 20, it would be 28 total hits, or if I missed 30, I would have 38 total hits...

T: Well, how many hits does it take to reach the target score?

H: It takes at least 8 hits to make 56

T: Well, can you show the relationship between the hits and the total score with symbols?

H: Teacher, I show as a symbol... I think of the triangle as 7 in my mind and 8 as a square. Then, teacher, I think of 5 as a circle and 6 as an X...

T: Well, can you show it in different ways?

H: Teacher... I can do an addition... I can do it by adding 8, but teacher, this will take too long.

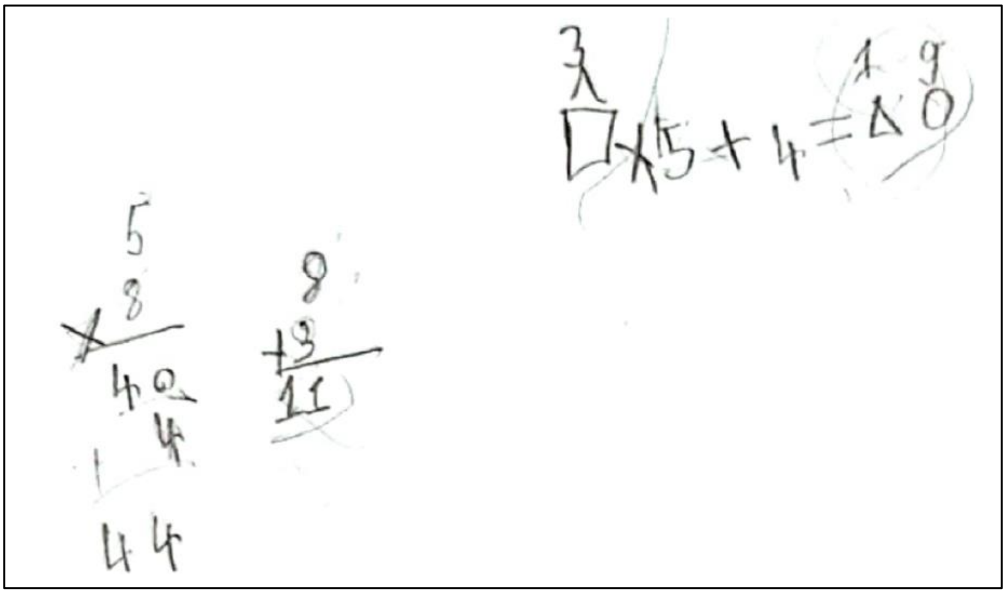


Figure 4.18. Harun’s Symbolic and Numerical Representation Form for Question 5 in the Game-Interview

Harun's solution for the fifth question is presented in Figure 4.18. While solving the fifth question with symbols, the student correctly wrote the equation indicating the function rule by using the symbols he determined separately for each number from 0 to 9. In addition, in this question and other questions in the second part, symbols were used only to indicate varying quantities, without any symbols for constant terms and coefficients.

4.3.4 Findings for Funda

Funda had some difficulty in adapting to the game and making accurate hits at first. But then she recovered and started to pass the levels in a shorter time and successfully. During the game and the transitions between levels, she correctly explained the values on the scoreboard, the minimum number of hits required to reach the target score, and the relationships between the number of hits and scores at some levels but had difficulty on other levels.

Table 4.20 Funda’s Findings for Game-Interview

Items	**Correctness**	**Strategy**	**Representation**
1a	0	O	TV
1b	0	RPG	TVN
1c	0	RPG	TVN
2a	0	SI	TVN
2b	1	FCS	TVNS
2c	0	FCS	TVNS
2d	0	O	VN

In Table 4.19, Funda's findings for the game-interview were shown. He answered all the questions apart from items 2b and 2c by using verbal, numeric, and tabular representations. In addition to these, symbolic and verbal representation forms were used in items 2b and 2c. The strategies used by the Funda were coded as RPG in items 1b and 1c, while it was coded as FCS in items 2b and 2c.

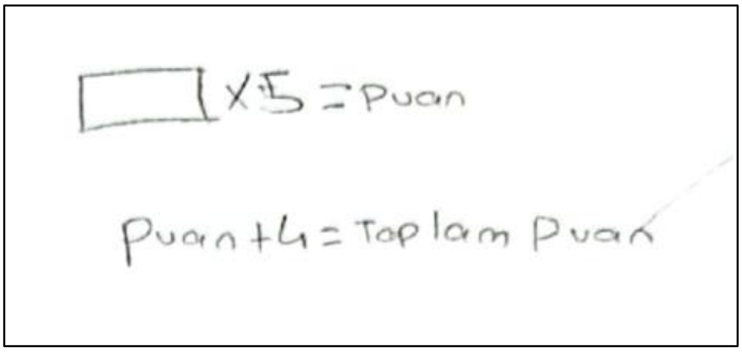


Figure 4.19. Funda’s Symbolic, Verbal and Numerical Representation forms for Question 5 in the Game-Interview

A part of Funda’s solution path for the fifth question is given in Figure 4.19. While expressing the number of hits with a box symbol, he wrote the other variables verbally and did not use any variables for constant terms or coefficients. The answers given by Funda while solving this question were as follows:

T: Can you show the relationship between the number of hits and the total score with symbols?

F: I do something like...

T: Can you explain to me what you're doing here?

F: First, I multiply the number of hits by 5 and found the score. I added this score to the initial score and found the score again. that is the last score.

T: What does the box mean here?

F: number of hits...

4.4 A General Summary of the Findings for Each Student

In this part of the chapter, a summary will be provided for each participant.

Zeynep's answers in the pre-test and pre-interview varied at different levels from RPP to FCW. After playing the game, the most usied levels were changed to FCW and FCS. While the highest level Zeynep could reach in the second question was CR, this level changed to FCW after interacting with the game. While Zeynep did not use variables in any way at the beginning and used symbols only for visualization purposes, after interacting with the game, she used symbols as variables correctly and meaningfully and wrote them in different ways. Similarly, for the second problem, while Zeynep did not use variables or symbols at the beginning, she managed to use the symbols as variables in a correct and meaningful way and write them in different ways after interacting with the game. This shows that the game-based learning activity had a positive effect on Zeynep and provided a significant improvement in the generalization and representation processes of functional relationships.

While the levels of Yavuz's answers in the first question varied, the highest level he could get was usually CR. However, after interacting with the game, this variety in his answers decreased and changed to the highest level of FPS he could get. While Yavuz's answer to the second question was CR, which was the highest level he could reach, these levels changed after his interaction with the game, and the highest level he could reach changed to FPS. While Yavuz did not answer the questions of representation with symbols or variables at first and left these questions unanswered, he started to use symbols meaningfully after his interaction with the game. He answered these questions using symbols in the post-test and the post-interview. This was one of the most important effects of interaction with the game on Yavuz. This shows that the game-based learning activity had a positive effect on Yavuz and that it provides a significant improvement in the generalization and representation processes of functional relationships.

While the levels of Harun's answers to the first question in the pre-test and pre-interview varied from the RPG level to the FCW level, which was the highest level he could get. After interacting with the game, in the post-test and post-interview, the highest level he could reach was FCW again, but the answers at this level showed intensity. While Harun answered the questions in the CR level, which was the most sophisticated level and frequently encountered in the second question, after interacting with the game, the highest level he could get increased to FPS. Harun tried to use the symbols in the pre-test and pre-interview but was coded in the other category because he expressed the relationships incorrectly or could not explain the symbols properly. While playing the game, he started to correctly express the relationships between the variables with symbols, so these answers were coded at the FPS level in the post-test and post-interview. This shows that after interacting with the game, Harun significantly improved his generalization and representation processes of functional relationships.

While Funda showed intensity at the RPG level as the highest level at the beginning, the answers at the FP level were frequently encountered as the highest level after interacting with the game, which showed that the game caused a partial rise for Funda. Although there was not much difference in the pre-test and the post-test in terms of generalization levels, Funda used symbols as variables during the interviews and could write the function rule correctly using symbols.

CHAPTER 5

DISCUSSION AND CONCLUSION

This study investigated the fifth-grade students' functional thinking processes in game-based learning and explored their ability to generalize and represent functional relationships. With this aim, four fifth-grade students' gaming processes and their responses to the problems in the FTT were analyzed. In this section, the findings of the study will be discussed, and also recommendations for future studies and implications will be presented.

5.1 Discussion

This section contains discussions of the present study's findings, divided into two sections. The focus of the first part is on students' generalization levels and representation processes of functional relationships and interpreting them based on the framework of Stephens et al. (2017) and the representation framework, which was compiled from different studies (Pinto & Canadas, 2021; Tanışlı, 2011; Urena et al., 2022). Changes in students' functional thinking processes after interaction with the game will be focused on in the second part.

5.1.1 Students' Generalization Levels and Representation Processes of Functional Relationships

In this part of the study, students' generalization levels and representation processes of functional relationships will be discussed based on the framework of Stephens et al. (2017) and based on the representation framework which was compiled from different studies as mentioned (e.g., Pinto and Canadas, 2021; Tanışlı, 2011; Urena et al., 2022). The Functional- emergent in variables and functional- condensed in variables levels into representations were adapted in this study as Functional Particular in symbols, Functional- emergent in variables or symbols, and Functional-condensed in variables or symbols (see Table 3.8). This change became an important distinction in terms of the analysis and findings of the study. The reason for this could be that while children generally did not have problems expressing the relationships between variables by performing numerical operations or verbally, they found it easy writing this relationship using symbols in comparison with using variables. Instead of using letters as variables, such as x, and y, it seemed to be more practical for students to use symbols as variables, such as box, star, and circle. In this way, they were able to express functional relationships more easily. That is to say, describing function rules with symbols was found to be an intermediate step for almost all students. Therefore, in this study, it was thought that the way of representation using symbols could be positioned as an intermediate step in the transitions between children's levels of expressing functional relationships verbally and with variables.

Blanton et al. (2011) mentioned that the concept of variable has different meanings depending on the context used. One of them is that it represents a fixed and unknown value. For a given equation to be true, that value must be equal to a constant number. In the National Mathematics curriculum (MoNE, 2018), our students are introduced to the concept of the unknown using symbols for the first time in the second grade. With the objective, M.2.1.2.2 " Students find the added number that is not given in the sum of two numbers," they are expected to find the value of the given number to provide equality in the addition process (MoNE, 2018, p. 32). Later, in the fourth grade, there is a related objective, M.4.1.5.7, "Students identify the missing value in one of the two mathematical expressions with equality between them and explain that the equality is achieved" (MoNE, 2018, p. 46). For this, students are given an equation, and a box symbol represents the unknown term. Then, they are expected to find the value that should replace this box symbol.

Another role Blanton et al. (2011) mentioned in their work is that for a given equation to be correct, that value must be equal to more than one number that satisfies the equation depending on the other quantity. In this study, students used symbols in this role when asked to explain the relationship between variables. This showed that students used these symbols as 'varying quantity' rather than 'unknown value' with which they were familiar through the curriculum from the earlier grades.

Parallel to Tanışlı's (2011) findings, in the beginning, students were first found to utilize a recursive strategy to examine the function tables, looking for a recursive pattern. She said that the students first recognized the dependent variable's values as a pattern without taking the independent variable into account, and then they focused on the difference between the pattern's successive values. As stated in other studies such as Lannin et al. (2006) and Stacey and MacGregor (2001), one of the students in this study, Funda, especially focused on the recursive pattern since the independent variable was growing by one and the dependent variable was aligned progressively in the tables. Funda answered both problems intensively at the RPG level in almost all implementations. For the first problem, she replied that “the number of tables increases by one and the number of people increases by two,” and for the second problem, ”the number of days increased by one and the number of leaves increased by three.” This shows that she generally looked down the columns and focused on only the difference between the terms, she was not aware of the other functional relationships such as covarational and correspondence.

Urena et al. (2022) examined the strategies and representations sixth-grade primary school students used to generalize functional relationships. Students used various strategies and representations, such as verbal, symbolical, or multiple, to generalize. The most widely used was the correspondence strategy which was coded as functional condensed in words (FCW) and functional condensed in variables (FCV) in this study. The most used strategy in this study was functional particular. Students were able to use symbolic as well as verbal representations while generalizing. They found that when the students were asked about the situation on the 100th day, although the number of no responses was high, most of the students answered using verbal and numeric representation forms. Also, they reported that some answered this situation for the n^{th} day, and they could generalize for the n^{th} day as well. In this study, when the students were asked about the number of people on the 100^{th} day of the first problem, which addressed a function rule of y =mx, they answered correctly both verbally and numerically by using the function rule in all implementations. In the second problem, which addressed a function rule of y =mx+n, when the 100^{th} day was asked, the students applied the function rule similarly in almost all implementations at a lower rate. This finding is consistent with earlier studies that show that in elementary school, students can employ verbal and numerical representations in general circumstances (e.g., Pinto & Canadas, 2021; Torres et al., 2019).

Another investigation of fifth-grade students' functional thinking processes was conducted by Akın (2020). According to findings of the Akın's (2020) study, the experimental group of students showed an improvement in their functional thinking processes. It was also revealed that although students tended to use recursive patterns to explain the relationships between the variables in the pre-test, experimental students were better able to establish covariational relationships and state the function rule using words and variables. She also pointed out that students had more success defining the y=2x relationship than the y=3x+2 relationship. At this point, similar findings are encountered in our study. It was determined that the students had more difficulty in generalizing and representing the y=2x+2 relations in the first question and the y=3x+4 relations in the second question compared to the y=2x relation. It was seen that the students experienced more difficulties in generalizing the patterns in the y=mx+n format. There are also different studies that obtained similar findings (e.g., Blanton et al., 2015; Stephens et al., 2015; Türkmen & Tanışlı, 2019).

After the interaction with the game, the answers coded as No Response in the pre-test disappeared, as the students could reason more about the relationships between quantities and increased their ability to express these relationships using words and variables. Similar findings were found in the study of Stephens et al. (2017). In their study, students had great difficulty in the generalization and representation of functional relationships before instruction. During early algebra lessons, students developed their abilities in identifying general recursive rules and expressing correspondence rules in both words and variables. As a difference in these findings, in this study, the students expressed the functional relationships by using symbolic representations. As consistent with studies in the elementary grades, some students remained at recursive and covariational reasoning levels in some of the questions (e.g., Confrey & Smith, 1994, 1995; Lannin et al., 2006).

Most studies emphasize that primary and secondary school students' functional thinking skills can be developed from an early age (e.g., Blanton & Kaput, 2004; Warren et al., 2006; 2008). It was stated that students showed significant improvements in defining functional relationships and understanding patterns after a teaching process. In addition, they could express the relationships between quantities using variables or words. If students can think functionally and use this understanding to develop algebraic generalizations, it will promote ease of reasoning for situations that depend on more complex algebraic thinking.

Studies also showed that students found it much easier to describe and generalize functional relationships orally than to give a formal written response (e.g., Warren & Cooper, 2008). Similarly, while the participants in this study had problems explaining the relationship between quantities in the tests properly, they could express these relationships more easily in the interviews and gave higher-level answers. This shows that students could express themselves better verbally than in writing.

5.1.2 Changes in Students' Functional Thinking Processes After Interaction with the Game

In this study, the students were found to jump a step in the level of generalization and representation by just playing the game without any other training in algebra or functional relationships. There was a remarkable improvement in almost all of the students when the pre-test and post-test or post-interview data were considered. At the beginning of the study, while the students gave answers at the lower levels of functional thinking, after interacting with the game, the student answers increased to the higher levels of functional thinking. This shows that the Functional Thinking Game had a positive effect on students' functional thinking processes.

The students were very enthusiastic about playing the game and focused on the game to pass the levels. While they were shooting, they tried to play more carefully by constantly looking at the scoreboard. In this way, the values on the scoreboard attracted their attention more, and this prompted them to think about functional relationships. Examples of dialogues during the game regarding this were given below:

...

Harun: Every time, it increases by 10 points... The numbers here are the same (total points and number of hits), but this (the number of shots) is always increasing. I missed a lot of shots… I have to do less to pass the level immediately...

...

Harun: I got 40 points in 25 shots. So, I hit 4 of them, and I missed 21 of them, so I had to take another shot. To get 50 points, I hit 5 of them out of 27 because each shot increased by 10 points; I missed 22 of them because I hit 5.

...

Yavuz: Here, in the beginning, 1 point is given, and each hit is 4 points, and the target is 25 points... Teacher, 6 x 4 = 24. Then I have to hit 6 shots (playing the game)

...

Yavuz: Total shot 7, number of hits 6, number of missed shots 1. Thus, I get 25 points.

Based on the dialogues above, it could be said that the students focused on the values in the scoreboard to pass the levels while playing the game and thus discovered the functional relationships. This might be because students were more willing and motivated to explore these relationships through playing game. This might be due to students' attention and interest when it comes to technological devices such as computers and tablets or games. In this way, their development in the process of discovering functional relationships within the game might have increased. In the literature, there were similar views stating that playing such game activities increased the students' interest in the lesson (e.g., Malone, 1981), they were more motivated to participate in the lesson and they developed more positive attitudes toward math learning (e.g., Ke, 2008; Malone, 1981), they developed deeper comprehension levels, logical thinking and problem-solving (e.g., Kirriemuir & McFarlane, 2004), and students' cognitive functions such as critical and strategic thinking enhanced (e.g., Allsop et al., 2013). In addition to these, much research which supports this claim has confirmed the teaching effectiveness of playing games (e.g., Dempsey et al., 1996; Ke & Grabowski, 2007; Rieber, 1996). In their study with fifth-grade children, Ke and Grabowski (2007) concluded that playing games effectively enhanced mathematics performance and encouraged positive math attitudes irrespective of student differences.

Van den Heuvel-Panhuizen et al. (2013) showed that playing a dynamic computer game supported by classroom discussions can promote algebraic reasoning in primary school classrooms. According to their findings, when the students' pre-test and post-test results were compared, performance was substantially improved among all grades. Unlike this study, the researchers gave the computer game to the children as homework and provided access to the duration of the interaction with the game and the answers given by the students through monitoring software. For this reason, they could not fully capture the cognitive processes of the students, such as the mental calculations they made, and they could not be sure whether the students received help from others while working online. In this study, students were observed individually while playing the game, and then one-on-one interviews were made to examine the way of defining the functional relationships in the game and then the ways of thinking about the problems asked in parallel with the game and the solutions. This allowed the researcher to have more information about students' cognitive processes.

Another study with similar results was conducted by Siew et al. (2016). They used a quasi-experimental approach to investigate the effects of an Android app, the DragonBox 12+, on algebraic thinking and attitudes toward algebra among eighth-grade students. A pre-test was administered to sixty eighth-grade students. The experimental group was then exposed to the DragonBox 12+ via smartphone or tablet, whereas the control group was taught algebra via traditional methods involving imitation and repetition. They applied teaching and learning sessions lasting 16 hours. At the end of the session, a post-test was implemented to evaluate students' algebraic thinking. At the conclusion of the 15-minute answering post-test session, both the control and experimental groups were given an algebra questionnaire. They found that learning algebra with DragonBox 12+ had a positive effect on the students' algebraic thinking. This research also demonstrated that DragonBox 12+ could assist students in developing a more positive attitude toward algebra learning. Furthermore, students exposed to DragonBox 12+ demonstrated greater confidence in algebra when compared to students learning using traditional methods. Similarly, in this research, learning algebra with a digital game had a positive impact on the students' functional thinking processes, and they demonstrated a positive attitude towards learning functional relationships. They were very enthusiastic about passing the levels while playing the game and solving the game-related questions. This motivated them even more to explore functional relationships. After the implementation, they said that they enjoyed this process very much and that they wanted to play such games with their friends in the classroom. In their study, each student had a tablet or smartphone, and they played the game alone. When they had difficulty, there was a tips button to get help for solving the problem at that level. The teachers had the role of guiding the session as a facilitator. Unlike that, in this study, each student played the game individually in an empty room on the computer, and while playing the game, the students were asked to think aloud and comment on their progress in the levels. So "I need 20 points to pass this level.” or “I had a total of 9 shots and missed 5 of them. Now I have 16 points, so I have to make one more hit to reach the target point.” The children talked about the game process and focused on the functional relationships in the game. Thanks to these self-reflections of the students, the researcher had the opportunity to observe the student's awareness of the game and their way of thinking, and thanks to the thinking aloud, the students were able to better focus on the game and the values in the scoreboard. At this point, important features that distinguishes this study from other studies were that students were asked to think aloud and self-reflect.

5.2 Implications and Recommendations

After this study, a significant improvement was observed in the functional thinking processes of the students after the interaction with the game. There has been an increase in the generalization levels of the students and the diversity in the representation forms such as symbols or words. Since this study aimed to examine the development of students' functional thinking processes in depth after their interaction with the game, it was studied with a small group and focused on a single algebra task. Future studies in this field can be conducted on different topics, with different grade levels of students and with more participants. Similarly, one type of game was designed and used in this study. The number and structure of the games can be diversified to increase the motivation and interest of the students. Future studies could also use the eye-tracking method to measure how often students look at the scoreboard or how often they follow the dartboard and measure their progress. In this way, the focus or attention process of the students can be followed.

In the National Mathematics Curriculum (MoNE, 2018), students have worked on number and shape patterns at different grade levels since the beginning of primary school. Students start to use unknown quantities while doing arithmetic operations in the second grade. In addition, they encounter the concept of the variable for the first time in the sixth grade. Afterward, the subject of equations is intensively covered in the seventh grade. In the eighth grade, there are subjects such as algebraic expressions and identities, linear equations, and inequalities where functional thinking is more dominant. In this study, students were found to be able to generalize functional relationships and use different representations to generalize after their interaction with the game. While doing these, they were observed to set up equations and write the function rule using words, and symbols. In the light of these, it was seen that students were able to discover functional relationships at an early age and create and use algebraic expressions and equations meaningfully. Therefore, it can be concluded that functional thinking can be introduced to students in early grades in the curriculum with games and activities similar to the game-based learning activity used in this study, and this can help students develop algebraic thinking. Therefore, curriculum developers can consider the results of this study from this point of view. In addition, teachers and prospective teachers can also prepare and implement similar activities that help students discover functional relationships and represent and generalize algebraic situations through interaction with the games.

The functional thinking game can be applied as a classroom activity for different objectives related to functional thinking. It can also be applied to different grade levels by varying the level of difficulty and content. In addition, digital game-supported lesson plans for each objective can be prepared and applied to students for a school year or a few months in order to spread it over a longer period and measure different achievements and processes. In addition, the researcher can form control and experimental group,s and the differences between these groups can be examined.

5.3 Limitations of the study

The study was carried out with a small group to allow each student to play individually, observe them individually, and examine their solutions and ways of thinking in depth. Thus, the limited number of participants can be a restriction in this study. Another limitation could be that the same interview protocol was used in the pre-and post-interviews. This could have increased students' awareness of the tasks. To eliminate this, all questions were structured as open-ended, and the duration between the two interviews was arranged to prevent remembering the problems.

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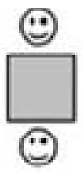
APPENDICES

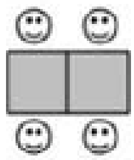
A. FUNCTIONAL THINKING TEST

1) Burak doğum günü partisine arkadaşlarını davet ediyor. Kare şeklindeki masaların etrafında her arkadaşı için oturacak bir yerin olduğundan emin olmak istiyor.

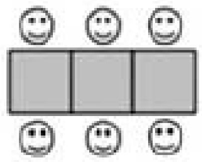
İlk masaya 2 kişi aşağıdaki şekilde gösterildiği gibi oturabiliyor:



Burak ilk masaya bir masa daha eklerse 4 kişi oturabilivor:



Eğer Burak ikinci masaya bir masa daha eklerse, 6



a) Aşağıdaki tabloyu Burak'ın farklı sayılardaki masalara kaçar kişi oturabileceğini düşünerek doldurunuz.

Masa Sayısı	**Kişi Sayısı**
1	
2	
3	
4	
5	
…	

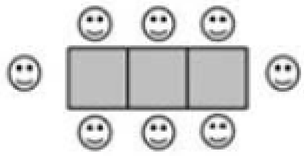
b) Tablodaki örüntüleri tanımlayınız.

c) Masa sayısı ile kişi sayısı arasındaki ilişkiyi açıklayan kuralı sözcüklerle yazınız.

d) Kutu, yıldız, daire gibi sembolleri veya harfleri kullanarak masa sayısı ile kişi sayısı arasındaki ilişkiyi açıklayan kuralı yazınız.

e) Eğer Burak'ın 100 masası varsa, kaç kişi oturabilir? Cevabınızı nasıl bulduğunuzu yazınız.

f) Burak, masanın başı ve sonundaki boş olan yerlere 2 kişi oturduğunda masaya daha çok kişi oturabileceğini fark ediyor. Örneğin aşağıdaki şekilde görüldüğü gibi Burak'ın 3 masası olduğunda, 8 kişi oturabiliyor.



Eğer Burak masanın yanlarına 2 kişi daha eklerse aşağıdaki tabloyu yeni durumda masalara kaçar kişi oturabileceğini düşünerek doldurunuz.

Masa Sayısı	**Kişi Sayısı**
1	
2	
3	
4	
5	
6	
7	

g) Tablodaki örüntüleri tanımlayınız.

h) Masa sayısı ile kişi sayısı arasındaki ilişkiyi açıklayan yeni kuralı sözcüklerle yazınız.

i) Kutu, yıldız, daire gibi sembolleri veya harfleri kullanarak masa sayısı ile kişi sayısı arasındaki yeni ilişkiyi açıklayan kuralı yazınız.

2) Ceren çiçekleri ve bitkileri çok sevdiği için bahçesine bir fidan dikmek istiyor. Bu fidan satın alındığında 4 yaprağı vardır. Dikildiği gün ve sonraki her gün fidanda 3 yeni yaprak filizleniyor.

a) Gün sayılarını ve toplam yaprak sayısını gösteren bir tablo yapınız.

b) Tablodaki örüntüleri tanımlayınız.

c) Gün sayısı ile yaprak sayısı arasındaki ilişkiyi açıklayan kuralı sözcüklerle yazınız.

d) Kutu, yıldız, daire gibi sembolleri veya harfleri kullanarak gün sayısı ile yaprak sayısı arasındaki ilişkiyi açıklayan kuralı yazınız.

e) 100. günün sonunda bu bitkinin kaç yaprağı olur? Cevabınızı nasıl bulduğunuzu yazınız.

B. INTERVIEW PROTOCOLS

Oyunun Birinci Bölümünü İzleyen Görüşme Soruları

- Oyunun ilk bölümünde puan artışları ve atış sayısı arasında nasıl bir ilişki vardır?
- Toplam puan ve isabetli atış arasındaki ilişkiyi tabloda gösteriniz.
- Tabloda hangi örüntüler vardır? Açıklayınız.
- Toplam puan ve isabetli atış arasındaki ilişkiyi tanımlayan kuralı sözcüklerle yazınız.
- Bu ilişkiyi tanımlayan kuralı kutu, yıldız, daire gibi sembolleri veya harfleri kullanarak ifade edebilir misin?

a) Her isabetli atış için 7 puan kazanan bir oyuncu minimum kaç atışta 56 puan hedefine ulaşır?

  - o Nasıl düşündüğünü açıklar mısın?
  - o Toplam puan ve isabetli atış arasındaki ilişkiyi tabloda gösteriniz.
  - o Tabloda hangi örüntüler vardır? Açıklayınız.
  - o Atış sayısı ile elde edilen puan arasındaki ilişkiyi açıklayan kural nedir?
  - o Bu ilişkiyi tanımlayan kuralı kutu, yıldız, daire gibi sembol veya harfleri kullanarak ifade edebilir misin?

a) Her isabetli atış için 5 puan kazanan bir oyuncu, 3 atış kaçırdıysa minimum kaç atışta 30 puan hedefine ulaşır?

  - o Nasıl düşündüğünü açıklar mısın?
  - o Toplam puan ve isabetli atış arasındaki ilişkiyi tabloda gösteriniz.

- Tabloda hangi örüntüler vardır? Açıklayınız.
- Atış sayısı ile elde edilen puan arasındaki ilişkiyi açıklayan kural nedir?
- Bu ilişkiyi tanımlayan kuralı kutu, yıldız, daire gibi sembol veya harfleri kullanarak ifade edebilir misin?

b) Oyuncu 10 atıştan dördünü kaçırmıştır. Toplamda 42 puan aldığına göre her isabetli atış kaç puandır?

- (Gidişata göre 5, 100 eklenebilir) 5 isabetli atış ve 5 ıskalama ile toplamda 5 puan almak için oyun kuralı ne olmalıdır?
- 100 isabetli atış ve 100 ıskalama ile toplamda 100 puan almak için oyun kuralı nedir?

Oyunun İkinci Bölümünü İzleyen Görüşme Soruları

- Oyunun ikinci bölümünde puan artışları, başlangıç puanı ve atış sayısı arasında nasıl bir ilişki vardır? Bu ilişkiyi farklı şekillerde ifade edebilir misin?
- Oyunun ikinci bölümünde puan artışları ve atış sayısı arasında nasıl bir ilişki vardır?
- Toplam puan ve isabetli atış arasındaki ilişkiyi tabloda gösteriniz.
- Tabloda hangi örüntüler vardır? Açıklayınız.
- Toplam puan ve isabetli atış arasındaki ilişkiyi tanımlayan kuralı sözcüklerle yazınız.
- Bu ilişkiyi tanımlayan kuralı kutu, yıldız, daire gibi sembolleri veya harfleri kullanarak ifade edebilir misin?

a) Oyuna 4 puanla başlayıp her isabetli atışta 7 puan kazanan bir oyuncu minimum kaç atışla 60 puana ulaşır?

  - Nasıl düşündüğünü açıklar mısın?
  - Atış sayısı ile elde edilen puan arasındaki ilişkiyi sözel olarak açıklayınız.
  - Bu ilişkiyi tanımlayan kuralı kutu, yıldız, daire gibi sembol veya harfleri kullanarak ifade edebilir misin?

b) Oyuna 4 puanla başlayan ve her isabetli atış için 5 puan kazanan bir oyuncu, 3 atış kaçırmıştır. Minimum kaç atışla 44 puan hedefine ulaşır?

  - Nasıl düşündüğünü açıklar mısın?
  - Atış sayısı ile elde edilen puan arasındaki ilişkiyi sözel olarak açıklayınız.
  - Bu ilişkiyi tanımlayan kuralı kutu, yıldız, daire gibi sembol veya harfleri kullanarak ifade edebilir misin?

c) Oyuna 7 puanla başlayan bir oyuncu 12 atıştan beşini kaçırmıştır. Toplamda 70 puan aldığına göre her isabetli atış kaç puandır?

  - Nasıl düşündüğünü açıklar mısın?
  - Atış sayısı ile elde edilen puan arasındaki ilişkiyi sözel olarak açıklayınız.
  - Bu ilişkiyi tanımlayan kuralı kutu, yıldız, daire gibi sembol veya harfleri kullanarak ifade edebilir misin?

C. APPROVAL OF THE UNIVERSITY HUMAN SUBJECTS ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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26 OCAK 2022

Konu : Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi : İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Işıl İşler BAYKAL

Danışmanlığını yürüttüğünüz Tuba ARSLANDAŞ'ın "Beşinci Sınıf Öğrencilerinin Fonksiyonel Düşünme Becerilerinin Oyun Temelli Bir Öğrenme Etkinliği ile İncelenmesi" başlıklı araştırmanız İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **0090-ODTUİAEK-2022** protokol numarası ile onaylanmıştır.

Saygılarımızla bilgilerinize sunarız.

Prof.Dr. Mine MISIRLISOY
İAEK Başkan

D. APPROVAL OF THE MINISTRY OF NATIONAL EDUCATION

T.C.
MARDİN VALİLİĞİ
İl Millî Eğitim Müdürlüğü

Sayı : E-63050228-605.01-45739838 15.03.2022
Konu : Araştırma Uygulama İzni

DAĞITIM YERLERİNE

a) Orta Doğu Teknik Üniversitesi Rektörlüğü Öğrenci İşleri Daire Başkanlığı 24/02/2022 tarih ve 54850036-044-E.284 sayılı yazısı.
b) Dicle Üniversitesi Rektörlüğü Öğrenci İşleri Daire Başkanlığı 01/03/2022 tarihli ve E-68508712-044-241102 sayılı yazısı.
c) Mardin Artuklu Üniversitesi Lisansüstü Eğitim Enstitüsü Müdürlüğünün 06/03/2022 tarih ve E-65966818-300-46951 sayılı yazısı.
d) Millî Eğitim Bakanlığının 21/01/2020 tarihli 2020/2 nolu Araştırma Uygulama İzinleri Genelgesi.
e) Valilik Makamının 12/03/2022 tarihli ve E-63050228-605.01-45558398 sayılı Oluru.

İlgi (a) yazıda, Matematik ve Fen Bilimleri Eğitimi Anabilim Dalı yüksek lisans programı 2085157 numaralı öğrencisi **Tuba ARSLANDAŞ "Beşinci Sınıf Öğrencilerinin Fonksiyonel Düşünme Becerilerinin Oyun Temelli Bir Öğrenme Etkinliği ile İncelenmesi"** başlıklı tez çalışması kapsamında Mardin ili Savur ilçesine bağlı Başkavak Ortaokulu, Başkavak İmam Hatip Ortaokulu 01/03/2022-20/06/2022 tarihleri arasında uygulama yapması ile ilgili evrakları incelenmiş olup;

İlgi (b) yazıda, Eğitim Bilimleri Enstitüsü Matematik Eğitimi Bilim Dalı tezli yüksek lisans programı 20978011 numaralı öğrencisi **Mehmet DEMİR "ACODESA metodu ile tasarlanan GeoGebra destekli öğrenme ortamında ortaokul öğrencilerinin üçgenler konusundaki matematiksel akıl yürütmelerinin incelenmesi"** başlıklı tez çalışması kapsamında Mardin ili Kızıltepe ilçesinde yer alan 24 Kasım Ortaokulunda 21/03/2022–28/12/2022 tarihleri arasında uygulama yapması ile ilgili evrakları incelenmiş olup;

İlgi (c) yazıda, Lisansüstü Eğitim Enstitüsü Müdürlüğü Eğitim Bilimleri Anabilim Dalı Başkanlığı Eğitim Programları ve Öğretim Programları Tezli Yüksek Lisans programı öğrencisi **Süleyman TEMUR "Ortaokul Öğrencilerinin Özgürlük Kavramına İlişkin Algılarının Sinektik Tekniğiyle İncelenmesi"** başlıklı tez çalışması kapsamında Mardin ili Kızıltepe ilçesine bağlı resmi ortaokul kurumlarında 14/03/2022-30/04/2022 tarihleri arasında uygulama yapması ile ilgili evrakları incelenmiş olup;

Türkiye Cumhuriyeti Anayasası Millî Eğitim Temel Kanunu ile Türk Millî Eğitiminin genel amaçlarına uygun olarak, 6698 sayılı Kişisel Verilerin Korunması Kanununa, yürürlükteki diğer tüm düzenlemelerde belirtilen hüküm esas ve amaçlara aykırılık teşkil etmeyecek şekilde, denetimleri ilgili ilçe millî eğitim müdürlükleri ve okul/kurum idaresinde olmak üzere, kurum faaliyetlerini aksatmadan, gönüllülük esasına dayalı olarak yapması ilgi (e) olurda uygun görülmüştür.

Bilgilerinizi rica ederim.

Murat DEMİR
Vali a.
İl Millî Eğitim Müdürü